MATHEMATICS-X

MODULE-4

INDEX

SIMILAR TRIANGLES

1.1 SIMILAR GEOMETRIC FIGURES

Two geometric figures which are same in shape, such that one is simply a copy of the other on a smaller scale or a larger scale, are called similar geometric figures. Two geometric figures are said to be similar if and only if they have the same shape but not necessarily the same size.

Note: Two congruent geometric figures are always similar but converse may or may not be true.

1.2 SIMILAR POLYGONS

Two polygons of the same number of sides are similar,if

(i) Their corresponding angles are equal and

(ii) Their corresponding sides are in proportion or their corresponding sides are in the same ratio

Note : The same ratio of the corresponding sides is referred to as the representative fraction or the scale factor for the polygons.

1.3 SIMILAR TRIANGLES

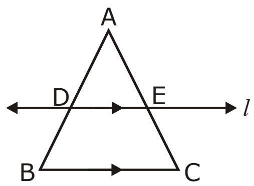
Two triangles are said to be similar, if

(i) Their corresponding angles are equal and

(ii) Their corresponding sides are in proportion (or are in the same ratio)

1.4 BASIC PROPORTIONALITY THEOREM (OR THALES THEOREM)

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.



If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

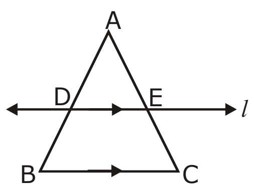
If in $\triangle ABC$, $l \parallel BC$, intersecting in D and E, then

(i) $\frac{AD}{DB} = \frac{AE}{EC}$ (ii) $\frac{AD}{AB} = \frac{AE}{AC}$ (iii) $\frac{DB}{AB} = \frac{EC}{AC}$

1.5 CONVERSE OF BASIC PROPORTIONALITY THEOREM

If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

i.e., In $\triangle ABC$, if l intersects AB in D and AC in E, such



that $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC

1.6 CRITERIA FOR SIMILARITY OF TRIANGLES

Two triangles are said to be similar, if

(i)Their corresponding angles are equal and

(ii)Their corresponding sides are in proportion (or are in the same ratio)

(i) AA or AAA Similarity Criterion :
If two angles of one triangle are equal to two corresponding angles of another triangle, then the triangles are similar.
If two angles of one triangle are respectively equal to the two angles of another triangle, then the third angles of the two triangles are necessarily equal, because the sum of three angles of a triangle is always 180°.

(ii) SAS Similarity criterion :
If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

OR

If two sides of a triangle are proportional to two corresponding sides of another triangle and the angles included between them are equal, then the triangles are similar.

(iii) SSS Similarity Criterion :
If in two triangles, sides of one triangle are proportional (or are in the same ratio) to the sides of the other triangle, then the triangles are similar.

Note : If $\Delta ABC \sim \Delta PQR$ by any one similarity criterion, then

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

i.e. A and P,B and Q,C and R are the corresponding vertices, also AB and PQ, BC and QR, CA and RP are the corresponding sides.

1.7 AREAS OF SIMILAR TRIANGLES

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Note :

(i) The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

(ii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding angle bisectors.

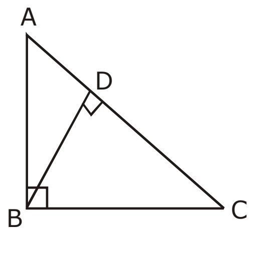
1.8 Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

1.9 Converse of pythagoras Theorem

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Note : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and similar to each other. i.e., If in ΔABC, $\angle B = 90°$ and $BD \perp AC$, then



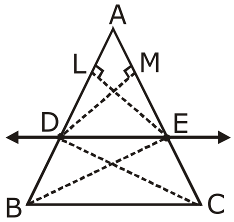
(i) $\Delta ADB \sim \Delta ABC$ (ii) $\Delta BDC \sim \Delta ABC$ (iii) $\Delta ADB \sim \Delta BDC$

1.10. Basic Proportionality theorem or thales theorem

If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given : ΔABC and a line 'l' parallel to BC intersects AB at D and AC at E as shown in figure.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$



Const. : Join BE and CD Draw EL $\perp$ AB and DM $\perp$ AC.

Proof : Area of ΔADE = $\frac{1}{2}$ × AD × EL ...(i) $\{\because$ Area of a $\Delta = \frac{1}{2}$ base× corresponding altitude$\}$

Area of ΔBDE = $\frac{1}{2}$ × DB × EL ....(ii) ,Now Dividing (i) and (ii), we have

$$\frac{\text{Area of}\Delta ADE}{\text{Area of }\Delta BDE} = \frac{\frac{1}{2}\times AD\times EL}{\frac{1}{2}\times DB\times EL} = \frac{AD}{DB} \qquad ...(iii)$$

Similarly,

$$\frac{\text{Are of }\Delta ADE}{\text{Area of }\Delta CDE} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} = \frac{AE}{EC} \qquad ...(iv)$$

Since, ΔBDE and ΔCDE are triangles on the same base DE and between the same parallels DE and BC

$\therefore$ Area of ΔBDE = Area of ΔCDE

From (iii) and (iv), we have $\frac{AD}{DB} = \frac{AE}{EC}$

Corollary : If in a ΔABC, a line DE || BC, intersects AB in D and AC in E, then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Proof :

(i) From the Basic Proportionality Theorem (B.P.T), we have taking reciprocals of both sides adding 1 on both sides]

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow \quad 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \quad \frac{AD+DB}{AD} = \frac{AE+EC}{AE}$$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) Again, from B.P.T., We have

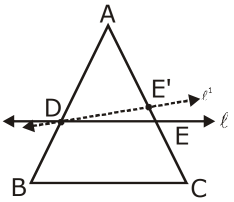
$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{DB}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \quad \frac{AD+DB}{DB} = \frac{AE+EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

1.11 Converse of Basic proportionality Theorem:
If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

Given : ΔABC and a line 'ℓ' intersects AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$

To Prove : Line l is parallel to BC or DE || BC



Proof : If possible, let the line ℓ is not parallel to BC. Through D, draw ℓ'|| BC intersecting AC in E' Now, by basic propotionlity theorem, we have

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \text{...(i)} \qquad \text{But} \qquad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{....(ii)}$$

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

Adding 1 to both sides, we have

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\Rightarrow \quad \frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C} \qquad \Rightarrow \quad \frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow EC = E'C$$

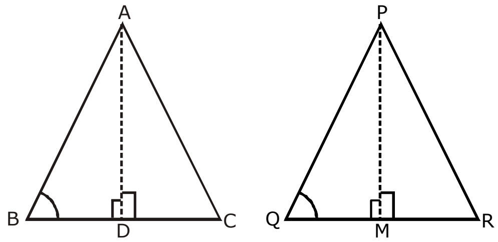
Which is only possible, if E and E' coincide. Hence, line ℓ is parallel to BC or DE || BC

1.12 The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given : $\Delta ABC \sim \Delta PQR$

To prove :

$$\frac{\text{Area of} \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$



Const : Draw AD $\perp$ BC and PM $\perp$ QR.

Proof: $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

Also, $\angle B = \angle Q$ [$\because$ corresponding angles of similar triangles are equal]

and $\angle ADB = \angle PMQ$ [each = 90°]

$$\Rightarrow \quad \Delta ADB \sim \Delta PMQ \quad \Rightarrow \frac{AD}{PM} = \frac{AB}{PQ}$$

From (i) and (ii), we have $\frac{AD}{PM} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

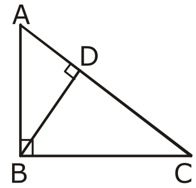
Now, $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM},$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC}{QR} \times \frac{AD}{PM} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$$

Also, from (i), we have $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Therefore, form (iv) and (v) we have $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

1.13 (Pythagoras' theorem) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: A ΔABC in which $\angle ABC = 90^\circ$.

To Prove: $AC^2 = AB^2 + BC^2$.

Consturction: Draw $BD \perp AC$.

Proof: In ΔADB and ΔABC, we have:

$\angle A = \angle A$ (common) $\angle ADB = \angle ABC$

$\therefore \quad \Delta ADB \sim \Delta ABC \quad \Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$

$\Rightarrow \quad AD \times AC = AB^2$ ...(i)

In ΔBDC and ΔABC, we have

$\angle C = \angle C$ (COMMON)

$\Delta BDC = \angle ABC$ [each equal to 90º]

$\therefore \quad \Delta BDC \sim \Delta ABC$ [by AA - similarity]

$\Rightarrow \quad \frac{DC}{BC} = \frac{BC}{AC} \quad \Rightarrow \quad DC \times AC = BC^2$ ...(ii)

From (i) and (ii), we get

$AD \times AC + DC \times AC = (AB^2 + BC^2)$

$\Rightarrow \quad (AD + DC) \times AC = (AB^2 + BC^2)$

$\Rightarrow \quad AC \times AC = (AB^2 + BC^2)$

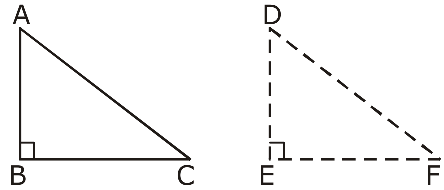
$\Rightarrow \quad AC^2 = (AB^2 + BC^2)$

1.14 (Converse of Pythagoras' theorem) In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

Given: A ΔABC in which $AC^2 = AB^2 + BC^2$.

To Prove: $\angle B = 90^\circ$.

Construction: Draw a ΔDEF such that $DE = AB$, $EF = BC$ and $\angle E = 90^\circ$.



Proof: In ΔDEF, we have : $\angle E = 90^\circ$. So, by Pythagora's theorem, we have :

$DF^2 = DE^2 + EF^2 \quad \Rightarrow \quad DF^2 = AB^2 + BC^2$ ......(i)

[$\because$ $DE = AB$ and $EF = BC$]

But, $AC^2 = AB^2 + BC^2$ ...... (ii)

[given]

From (i) and (ii), we get:

$AC^2 = DF^2 \quad \Rightarrow \quad AC = DF$.

Now, in ΔABC and ΔDEF, we have

$AB = DE$, $BC = EF$ and $AC = DF$ $\quad \therefore \quad \Delta ABC \cong \Delta DEF$

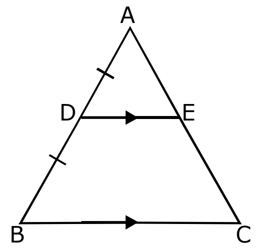
Hence, $\angle B = \angle E = 90^\circ$.

SOLVED PROBLEMS

Ex.1 Prove that the line drawn from the mid-point of one side of a triangle, parallel to another side, bisects the third side.

Sol. ΔABC in which D is the mid-point of side AB and the line DE is drawn parallel to BC, meeting AC in E.

To Prove : E is the mid-point of AC i.e, AE = EC.



Proof : In ΔABC, we have DE || BC.

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ ..... (i)

But D is the mid-point of AB.

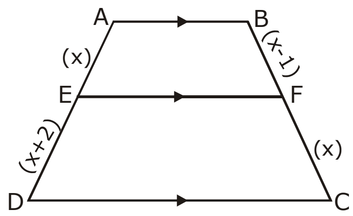
$\therefore \quad AD = DB \Rightarrow \frac{AD}{DB} = 1$ ..... (ii)

From (i) and (ii), we get

$\frac{AE}{EC} = 1 \Rightarrow AE = EC$

Hence, E is the mid-point of AC.

Ex.2 In figure if, EF || DC || AB, find x.



Sol. $\therefore \quad EF \parallel DC \parallel AB \therefore \frac{AE}{ED} = \frac{BF}{FC}$

$\Rightarrow \quad \frac{x}{x+2} = \frac{x-1}{x}$

$\Rightarrow \quad x^2 = (x + 2)(x - 1)$

$\Rightarrow \quad x^2 = x^2 + x - 2 \Rightarrow x = 2$ units

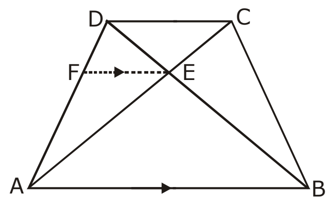
Ex.3 If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Sol. A quadrilateral ABCD whose diagonals AC and BD intersect at E such that $\frac{DE}{EB} = \frac{CE}{EA}$.

To prove : ABCD is a trapezium.

Construction : Draw FE || AB, metting AD in F.

Proof : In ΔABD, we have FE || AB.



$\therefore \quad \frac{DF}{FA} = \frac{DE}{EB}$ ... (i)

But $\frac{DE}{EB} = \frac{CE}{EA}$ ... (ii)

From (i) and (ii), we get

$\frac{DF}{FA} = \frac{CE}{EA}$

which means in ΔACD, E and F are points on AC and AD respectively such that

$\frac{DF}{FA} = \frac{CE}{EA}$

$\Rightarrow$ FE || DC ... (iii) [by converse of B.P.T.]

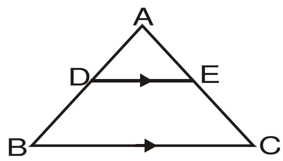
But, FE || AB ... (iv) [by construction]

From (iii) and (iv), we get : AB || DC.

Hence, **ABCD is a trapezium**.

Ex.4 In the adjoining figure, DE||BC.

(i) If AD = 3.4 cm, AB = 8.5 cm and AC = 13.5 cm, find AE.



(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and AC = 9.6 cm, find AE.

Sol. (i) Since DE||BC, we have $\frac{AD}{AB} = \frac{AE}{AC}$

$\therefore \quad \frac{3.4}{8.5} = \frac{AE}{13.5} \Rightarrow \frac{3.4 \times 13.5}{8.5} = 5.4$

Hence, AE = 5.4 cm.

(ii) Since DE||BC, we have $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \quad \frac{AE}{EC} = \frac{3}{5} \left[\because \frac{AD}{DB} = \frac{3}{5} \text{ (Given)}\right]$

Let AE = x cm. Then, EC = (AC – AE) = (9.6 – x) cm.

$\therefore \quad \frac{x}{9.6 - x} = \frac{3}{5} \Rightarrow 5x = 3(9.6 - x)$

$\Rightarrow \quad 5x = 28.8 - 3x \Rightarrow 8x = 28.8 \Rightarrow x = 3.6.$

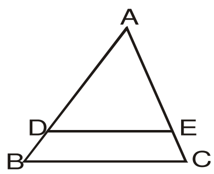
$\therefore \quad$ AE = 3.6 cm.

Ex.5 In the adjoining figure, AD = 5.6 cm, AB = 8.4 cm, AE = 3.8 cm and AC = 5.7 cm. Show that DE||BC.

Sol. We have, AD = 5.6 cm, DB = (AB – AD) = (8.4 – 5.6) cm = 2.8 cm.

AE = 3.8 cm, EC = (AC – AE) = (5.7 – 3.8) cm = 1.9 cm.

$\therefore \quad \frac{AD}{DB} = \frac{5.6}{2.8} = \frac{2}{1}$ and $\frac{AE}{EC} = \frac{3.8}{1.9} = \frac{2}{1}$



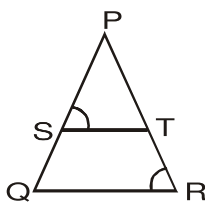
Thus, $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \quad$ DE divides AB and AC proportionally.

Hence, DE||BC

Ex.6 In fig, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

Sol. It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$



So, ST||QR [Theorem]

Therefore, $\angle PST = \angle PQR$ [Corresponding angles] - (1)

Also, it is given that

$\angle PST = \angle PRQ$ (2)

So, $\angle PRQ = \angle PQR$ [From 1 and 2]

Therefore PQ = PR

[Sides opposite the equal angles]

i.e., PQR is an isosceles triangle.

Ex.7 Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e., in the same ratio).

OR

ABCD is a trapezium with DC||AB. E and F are points on AD and BC respectively such that EF||AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$

Sol. We are given trapezium ABCD.

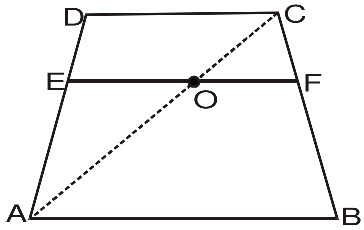
CD||BA

EF||AB and CD both

We join AC.

It mets EF at O.

In ΔACD, OE||CD



$$\Rightarrow \quad \frac{AO}{OC} = \frac{AE}{ED} \quad \text{...(i)}$$

(Basic Proportionality Theorem)

In ΔCAB, OF||AB

$$\Rightarrow \quad \frac{CO}{OA} = \frac{CF}{FB} \quad \text{[B.P.T]}$$

$$\Rightarrow \quad \frac{AO}{OC} = \frac{BF}{FC} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence, proved.

Ex.8 Any point X inside ΔDEF is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meeting XE at Q and QR is drawn parallel to EF meeting XF in R. Prove that PR || DF.

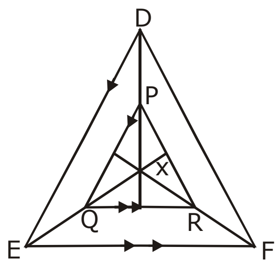
Sol. **Given :** In figure PQ || DE and QR || EF.

To Prove : PR || DF.

Proof : In ΔXED; PQ || DE.

$$\therefore \quad \frac{XP}{PD} = \frac{XQ}{QE} \quad \text{... (i)} \qquad \text{[by B.P.T.]}$$

Also, in ΔXEF, QR || EF $\quad \therefore$ We have



$$\frac{XQ}{QE} = \frac{XR}{RF} \quad \text{... (ii)} \qquad \text{[by B.P.T.]}$$

From (i) and (ii), we have

$$\frac{XP}{PD} = \frac{XR}{RF}$$

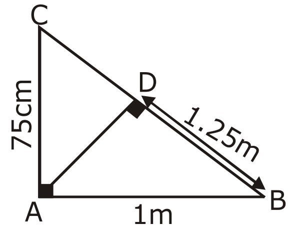
Thus, in ΔXFD, R and P are points dividing sides XF and XD in the same ratio.

Therefore, by converse of B.P.T.,we have **PR||DF**.

Ex.9 In the given figure, $\angle CAB = 90^o$ and $AD \perp BC$. If AC = 75 cm. AB = 1m and BD = 1.25 m, find AD.

Sol. $\because$ In a ΔABC, $\angle A = 90^o$ and $AD \perp BC$, where D is on BC.

$\therefore \quad \Delta BAC \sim \Delta BDA$



$$\Rightarrow \quad \frac{BA}{BD} = \frac{AC}{DA} = \frac{BC}{BA}$$

$$\Rightarrow \quad \frac{100}{125} = \frac{75}{DA} = \frac{BC}{BA} \quad [\because AB = 1m \;\; 100cm$$

and BD = 125 cm

$$\Rightarrow \quad \frac{100}{125} = \frac{75}{DA} \Rightarrow DA = \left(\frac{125 \times 75}{100}\right)cm$$

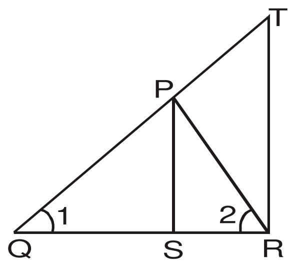
AD = 93.75 cm

Ex.10 In fig, $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.

Sol. $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ PR = PQ ...(i)

(Sides opposite to equal angles in ΔQRP)



Also $\frac{QT}{PR} = \frac{QR}{QS}$ (Given) ...(ii)

Form (i) and (ii), we have

$$\frac{QT}{PR} = \frac{QR}{QS} \Rightarrow \frac{QP}{QT} = \frac{QS}{QR} \quad ...(iii)$$

Now, in triangles PQR and TQR, we have

$\angle PQS = \angle TQR$ (each = $\angle 1$)

and $\frac{PQ}{TQ} = \frac{QS}{QR}$ (from (3))

$\Rightarrow \quad \Delta PQS \sim \Delta TQR$ (SAS Similarity)

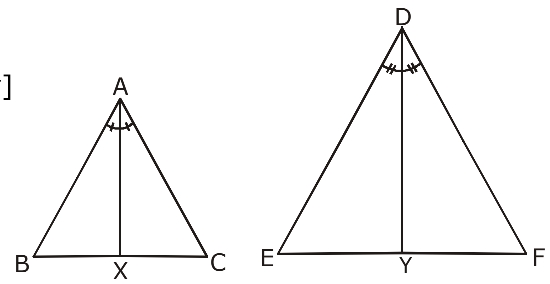
Ex.11 If two triangles are equiangular, prove that the ratio of corresponding sides is the same as the ratio of the corresponding angle bisector segments.

Sol. **Given :** Two Δs ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; and AX, DY are the bisectors of A and D respectively.

To porve : $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AX}{DY}$

Proof : Since, equiangular traingels are similar [by AA-similarity]

We have : $\Delta ABC \sim \Delta DEF$



$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad ..... (i)$$

Now, $\angle A = \angle D \Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle D$

$\Rightarrow \quad \angle BAX = \angle EDY$

Thus, in Δs ABX and DEY, we have : $\angle BAX = \angle EDY$ [proved]

and $\angle B = \angle E$ [given]

$\therefore \quad \Delta ABX \sim \Delta DEY$ [by A.A. similarity]

$$\therefore \quad \frac{AB}{DE} = \frac{AX}{DY} \quad .... (ii)$$

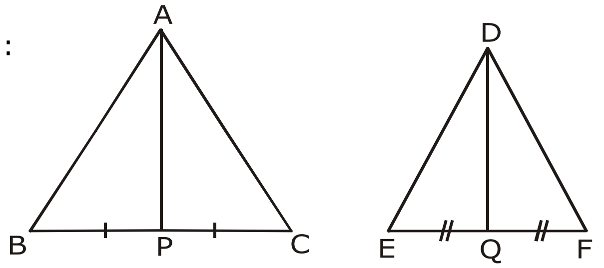
From (i) and (ii), we get :

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AX}{DY}$$

Ex.12 If two triangles are equiangular, prove that the ratio of the corresponding sides is the same as the ratio of the corresponding medians.

Sol. **Given :** Two Δs ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; AP and DQ are the medians.

To prove : $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AP}{DQ}$



Proof : Since, equiangular traingles are similar we have :

$\Delta ABC \sim \Delta DEF$ [by A.A. similarity]

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

But, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$ [$\because$ BC = 2BP, EF = 2EQ]

Now, in Ds ABP and DEQ, we have

$\frac{AB}{DE} = \frac{BP}{EQ}$ and B = E [given]

$\therefore$ $\Delta ABP \sim \Delta DEQ$ [byS.A.S. similarity]

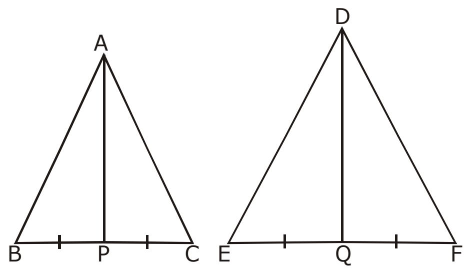
$\therefore \frac{AB}{DE} = \frac{AP}{DQ}$

From (i) and (ii), we get :

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AP}{PQ}$

Ex.13 If two sides and a median bisecting one of these sides of a triangle are respectively proportioanl to the two sides and the corresponding median of another triangle then the triangles are similar.

Sol. **Given :** ΔABC and ΔDEF in which AP and DQ are the medians are such that



$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$

To prove : $\Delta ABC \sim \Delta DEF$

Proof : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$ [given] .... (i)

$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{AP}{DQ}$ [note this step]

$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ}$ $\left(\because \frac{1}{2}BC = BP, \frac{1}{2}EF = EQ\right)$

$\Rightarrow$ $\Delta ABP \sim \Delta DEQ$ [by S.S.S. similarity]

$\Rightarrow$ $\angle B = \angle E$ ... (ii)

Now, in Δs ABC and DEF, we have

$\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle B = \angle E$ [from (i) and (ii)]

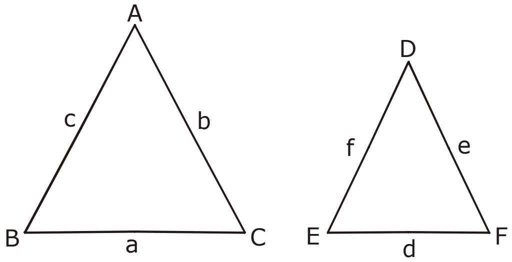
$\therefore$ $\Delta ABC \sim \Delta DEF$ [by S.A.S. similarity]

Ex.14 Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their correspoding sides.

Sol. $\Delta ABC \sim \Delta DEF$

To Prove :

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



Proof: Let BC = a, AC = b, AB = c,

EF = d, DF = e, DE = f

$\because \quad \Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k \text{ (say)} \qquad .... \text{(i)}$

or $\quad \frac{c}{f} = \frac{a}{d} = \frac{b}{e} = k \qquad .... \text{(ii)}$

$\therefore \quad c = fk, \ a = dk, \ b = ek$

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB + BC + AC}{DE + EF + DF}$$

[perimeter of a trianlge= sum of its sides]

$$\frac{c + a + b}{f + d + e} = \frac{fk + dk + ek}{f + d + e} \qquad \text{[using (ii)]}$$

$$= \frac{k(f + d + e)}{(f + d + e)} = k$$

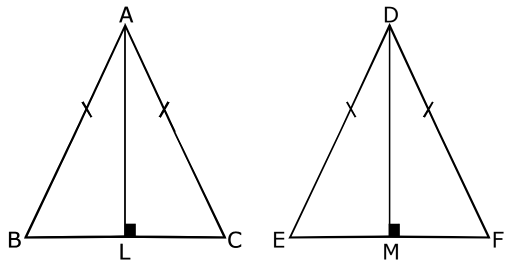
From (i) and (iii), we get

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Ex.15 Two isosceles triangles have equal vertical angles and their areas are in the same ratio 16 : 25. Find the ratio of their corresponding heights.

Sol. **Given :** Δs ABC and DEF such that AB = AC, DE = DF, $\angle A = \angle D$.

and $\quad \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{16}{25}$



To Determine : $\frac{AL}{DM} = ?$

Proof : AB = AC, DE = DF

$\Rightarrow \quad \frac{AB}{AC} = 1 \text{ and } \frac{DE}{DF} = 1$

$\Rightarrow \quad \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

Thus, in Δs ABC and DEF, we have

$\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$ [given]

$\therefore \quad \Delta ABC \sim \Delta DEF \quad$ [by S.A.S. similarity]

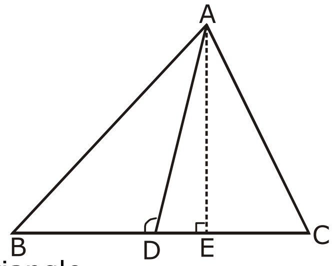
$\Rightarrow \quad \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{AL^2}{DM^2}$

$\Rightarrow \quad \frac{16}{25} = \frac{AL^2}{DM^2} \quad \left(\because \frac{\text{area }(\Delta ABC)}{\text{area }(\Delta DEF)} = \frac{16}{25} \right)$

$\Rightarrow \quad \frac{AL}{DM} = \frac{4}{5}$

Ex.16 Prove that in any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side.

Sol. **Given :** A ΔABC iin which AD is a median.



To prove : $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Const. : Draw $AE \perp BC$.

Proof : Since, $\angle AED = 90^o$. Therefore, in ΔADE, we have

$\angle ADE < 90^o \Rightarrow \angle ADB > 90^o$

Thus, ΔADB is an obtuse-angled triangle and ΔADC is an acute angled triangle.

Now, ΔABD is obtuse-angled at D and

$AE \perp BD$ produced.

we have $AB^2 = AD^2 + BD^2 + 2BD.DE$ ... (i)

Again, ΔACD is acute-angled at D and

$AE \perp CD$. we have

$AC^2 = AD^2 + DC^2 - 2DC.DE$

$\Rightarrow$ $AC^2 = AD^2 + BD^2 - 2BD.DE$ .... (ii)

[$\because$ CD = BD]

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow$ $AB^2 + AC^2$

$$= 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]\left[\because BD = \frac{BC}{2}\right]$$

$\Rightarrow$ $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

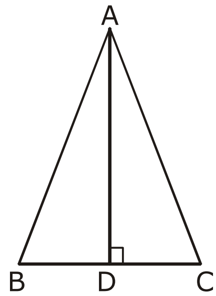
or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Ex.17 Prove that three times the square on any side of an equilateral triangle is equal to four times the square on the altitude.

Sol. **Given :** An equilateral ΔABC and $AD \perp BC$

To prove : $3AB^2 = 4AD^2$

Proof : We know that in an equilateral triangle perpendicular from a vertex bisect the base.



$\therefore$ $BD = DC = \frac{1}{2}BC$

Since, ΔADB is a right-triangle, right-angled at D.

$\therefore$ $AB^2 = AD^2 + BD^2$

[By Pythagoras Theorem]

$\Rightarrow$ $AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$

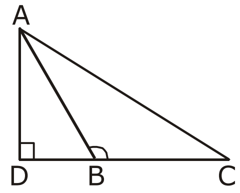
$\Rightarrow$ $AB^2 = AD^2 + \frac{BC^2}{4}$

$\Rightarrow$ $AB^2 = AD^2 + \frac{AB^2}{4}$ [$\because$ BC = AB]

$\Rightarrow$ $\frac{3}{4}AB^2 = AD^2 \Rightarrow 3AB^2 = 4AD^2$

Ex.18 In a ΔABC, $\angle ABC > 90^o$ and $AD \perp$ (CB produced). Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

Sol. **Given:** A ΔABC in which $\angle ABC > 90^o$ and $AD \perp$ (CB produced).



To Prove: $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

Proof: In ΔABD, $\angle ADB = 90^o$

$\therefore \quad AB^2 = AD^2 + BD^2$ ......(i)

[by Pythagoras' theorem]

In ΔADC, $\angle ADC = 90^o$

$\therefore \quad AC^2 = AD^2 + CD^2$

[by Pythagora's theorem]

$= AD^2 + (BC + BD)^2$

[$\because CD = (BC + BD)$]

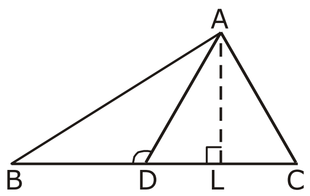
$= AD^2 + (BC^2 + BD^2 + 2BC \cdot BD)$

$= (AD^2 + BD^2) + BC^2 + 2BC \cdot BD$

$= (AB^2 + BC^2 + 2BC \cdot BD)$ [using (i)].

Ex.19 In ΔABC, if AD is the median, the prove that $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

Sol. **Given:** A ΔABC in which AD is the median.



To Prove: $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

Construction: Draw $AL \perp BC$.

Proof: In ΔALD, $\angle ALD = 90^o$

$\therefore \quad \angle ADL < 90^o$ and therefore,

$\angle ADB > 90^o$.

Thus, in ΔADB, $\angle ADB > 90^o$ and $AL \perp$ (BD produced).

$\therefore \quad AB^2 = AD^2 + BD^2 + 2BD \cdot DL$ ...(i)

In ΔADC, $\angle ADC < 90^o$ and $AL \perp DC$.

$\therefore \quad AC^2 = AD^2 + CD^2 - 2CD \cdot DL$

$\Rightarrow \quad AC^2 = AD^2 + BD^2 - 2BD \cdot DL$ ...(ii)

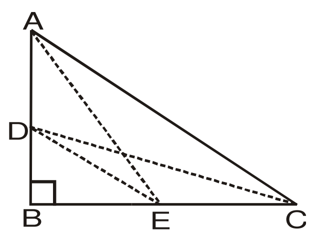
[$\because CD = BD$]

Adding (i) and (ii), we get :

$(AB^2 + AC^2) = 2(AD^2 + BD^2)$

Ex.20 In the given figure, $\angle B = 90°$. D and E are any points on AB and BC respectively.

Prove that : $AE^2 + CD^2 = AC^2 + DE^2$.



Sol. In ΔABE, $\angle B = 90°$

$\therefore \quad AE^2 = AB^2 + BE^2$ ...(i)

In ΔDBC, $\angle B = 90°$.

$\therefore \quad CD^2 = BD^2 + BC^2$ ...(ii)

Adding (i) and (ii), we get :

$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$

$= AC^2 + DE^2$ [By Pythagoras Theorem]

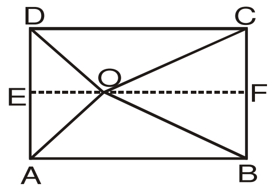
Hence, $AE^2 + CD^2 = AC^2 + DE^2$.

Ex.21 A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D.

Prove that: $OA^2 + OC^2 = OB^2 + OD^2$.

Sol. Through O, draw EF||AB. Then, ABFE is a rectangle.

In right triangles OEA and OFC, we have:



$OA^2 = OE^2 + AE^2$

$OC^2 = OF^2 + CF^2$

$\therefore$ $OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ ... (i)

Again, in right triangles OFB and OED, we have :

$OB^2 = OF^2 + BF^2$

$OD^2 = OE^2 + DE^2$

$\therefore$ $OB^2 + OD^2 = OF^2 + OE^2 + BF^2 + DE^2$

$= OE^2 + OF^2 + AE^2 + CF^2$ ...(ii) [$\because$ BF = AE & DE = CF]

From (i) and (ii), we get

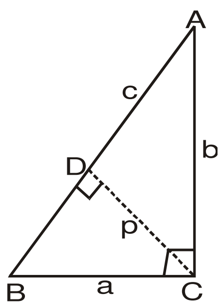
$OA^2 + OC^2 = OB^2 + OD^2$.

Ex22 In the given figure, ΔABC is right-angled at C.

Let BC = a, CA = b, AB = c and CD = p, where $CD \perp AB$.

Prove that: (i) cp = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol. (i) Area of $\Delta ABC = \frac{1}{2}$ AB × CD $= \frac{1}{2}$ cp.



Also, area of $\Delta ABC = \frac{1}{2}$ BC × AC $= \frac{1}{2}$ ab.

$\therefore \frac{1}{2}$ cp $= \frac{1}{2}$ ab. $\Rightarrow$ cp = ab

(ii) $cp = ab \Rightarrow p = \frac{ab}{c}$

$\Rightarrow$ $p^2 = \frac{a^2b^2}{c^2}$

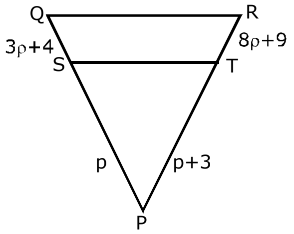
$\Rightarrow$ $\frac{1}{p^2} = \frac{c^2}{a^2b^2} = \frac{a^2+b^2}{a^2b^2}$ [$\because c^2 = a^2 + b^2$]

$\Rightarrow$ $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

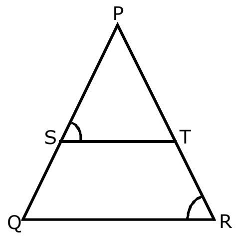
EXERCISE - I UNSOLVED PROBLEM

Q.1 If D and E are the point on the sides AB and AC of a triangle ABC respectively, such that AD = 6 cm, BD = 9 cm, AE = 8 and EC = 12 cm. Show that DE || BC.

Q.2 What value (s) of p will make ST || QR in the given Figure?



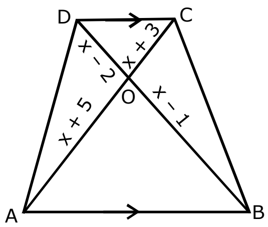
Q.3 In the adjoing in figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



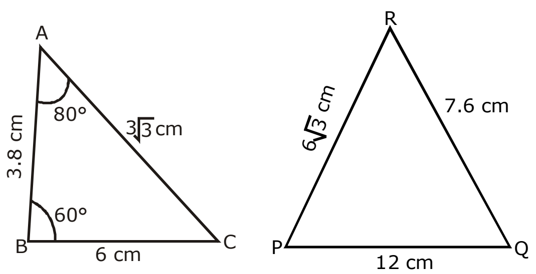
Q.4 E is the mid-point of side QR of a ΔPQR. D is the mid-point of PE when produced meets PR in G. Prove that PG = $\frac{1}{3}$ PR.

Q.5 ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$

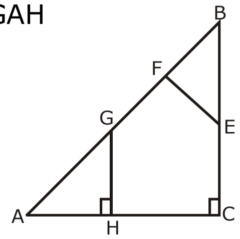
Q.6 In the given figure, if AB || DC, find the value of x.



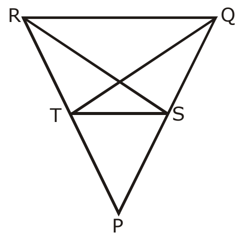
Q.7 Observe the given figure and then find $\angle P$.



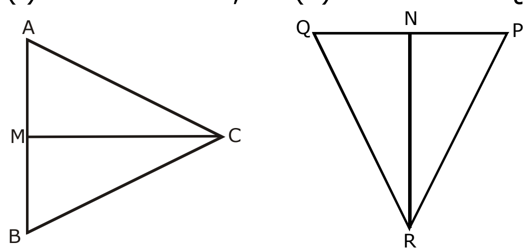
Q.8 In the given figure ΔABC is right-angled at C. E is any point on BC and EF $\perp$ AB. G is any point on AB such that GH $\perp$ AC. Prove that $\Delta BEF \sim \Delta GAH$



Q.9 In the given figure, $\Delta PRS \cong \Delta PQT$; prove that $\Delta PQR \sim \Delta PST$

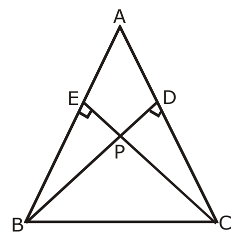


Q.10 In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, prove that :
(i) $\Delta AMC \sim \Delta PNR$, (ii) $\Delta CMB \sim \Delta RNQ$.



Q.11 D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that CA^2 = CB. CD.

Q.12 In the given figure, considering triangles BEP and CPD, prove that :
(i) In ΔABC if $\angle A$ is acute BD and CE are perpendicular on AC and AB respectively prove that AB × AE = AC × AD.
(ii) BP × PD = EP × PC



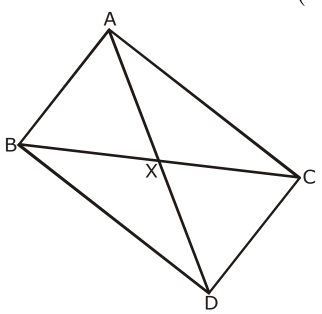
Q.13 A vertical stick 30 cm long casts a shadow 20 cm long on the ground. At the same time, a tower casts a shadow 40 m long on the ground. Determine the height of the tower.

Q.14 A boy of height 120 cm is walking away from the base of a lamp-post at a speed of 87m/minute. If the lamp-post is 36 m above the ground, find the length of his shadow after 3 minutes.

Q.15 ΔABC and ΔDEF are two similar triangles, BC = 3 cm, EF = 4 cm and area of ΔABC is 54 cm^2. Determine the area of ΔDEF.

Q.16 Areas of two similar triangles are 81 cm^2 and 49 cm^2. If the altitude of the first triangle is 6.3 cm. Find the corresponding altitude of the other.

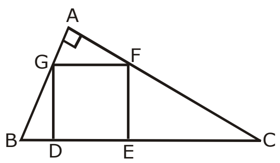
Q.17 In the given figure, ΔABC and ΔDBC are on the same base BC. Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AX}{DX}$.



Q.18 In the adjoining figure, DEFG is a square and $\angle BAC = 90°$. Prove that

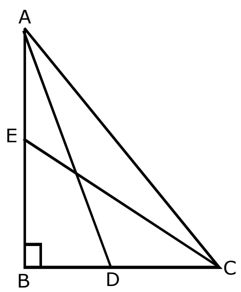
(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFG$

(iii) $\Delta DBG \sim \Delta EFG$ (iv) $DE^2 = BD \times EC$.



Q.19 D,E,F are respectively the mid-points of the sides AB,BC and CA of ΔABC. Find the ratios of the areas of ΔDEF and ΔABC.

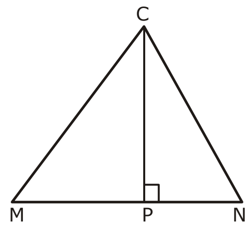
Q.20 In the given figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\dfrac{3\sqrt{5}}{2}$ cm, find the length of CE.



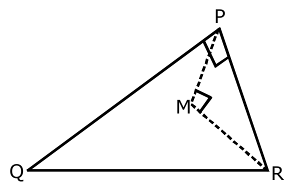
Q.21 A point O, in the interior of a rectangle ABCD, is joined with each of vertices A,B,C and D. Prove that: $OB^2 + OD^2 = OC^2 + OA^2$

Q.22 In the given figure LP $\perp$ MN. Prove that :

$LM^2 = MN^2 + LN^2 - 2.\ MN.\ PN$



Q.23 Using pythagoras theorem, find the area of ΔPQR, if $\angle P = \angle M = 90°$, PM = 6cm, MR = 8cm and QR = 26 cm.



Q.24 In a right-angled ΔABC, $\angle B = 90°$. If D is the mid-point of BC. Prove that $AC^2 = 4AD^2 - 3AB^2$.

Q.25 In the adjoining figure, PQR is a right triangle, right-angled at Q. Prove that:

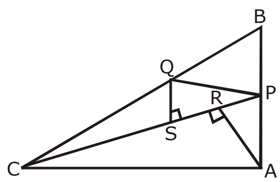
$\Delta PQR \sim \Delta PST$

Q.26 In ΔABC, if AD is the median, then show that $AB^2 + AC^2 = 2AD^2 + 2BD^2$.

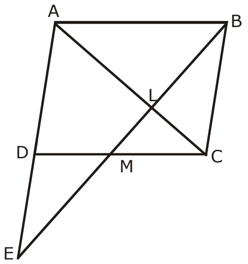
Q.27 In the given figure, P is a point on AB such that AP : PB = 4 : 3 PQ is parallel to AC.

(i) Find PQ : AC

(ii) In ΔARC, $\angle ARC = 90°$ and in ΔPQS, $\angle PSQ = 90°$, QS = 6cm, calculate the length of AR.



Q.28 In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM when joined meets AC in L and AD produced in E. Prove that EL = 2BL.



Q.29 In a right ΔABC, $\angle C = 90°$. P and Q are point on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that

(i) $9\,AQ^2 = 9AC^2 + 4BC^2$

(ii) $9\,BP^2 = 9BC^2 + 4\,AC^2$

(iii) $9\,(AQ^2 + BP^2) = 13\,AB^2$

Q.30 In a right triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x.

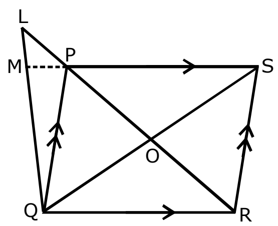
(i)Determine the value of x in terms of a and b.

(ii) Show that : $\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Q.31 In an acute angled ΔABC, AD is the median in it. Prove that:

$4AD^2 + BC^2 = 2AB^2 + 2AC^2$.

Q.32 In the adjoining figure, PQRS is a parallelogram. Diagonals PR and QS intersect at O.L is a point on the diagonal PR, such that $\frac{LP}{PO} = \frac{1}{2}$. QL meets SP produced at M. Find



(i) $\frac{LM}{MQ}$ (ii) $\frac{ar(\Delta LMP)}{ar(\Delta LQR)}$

(iii) $\frac{ar(\Delta LMP)}{ar(trap.\,MQRP)}$

Q.33 In ΔABC, DE || BC, such that $\frac{AD}{DB} = 5 : 4$.

Find

(i) DE : BC (ii) DO : DC

(iii) ar (ΔDOE) : ar (ΔDCE)

Q.34 In ΔABC, AD $\perp$ BC such that $AD^2 = BD \cdot CD$. Prove that ΔABC is right-angled at A.

Q.35 In the figure given below, ΔPQR is right-angled at Q and the points S and T trisect the side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Q.36 In ΔABC, AD $\perp$ BC and BD = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

Q.37 In ΔABC, $\angle B = 90°$ and D is the midpoint of BC. Prove that $AC^2 = AD^2 + 3CD^2$.

Q.38 ΔABC is right-angled at B and D is the midpoint of BC. Prove that $AC^2 = (4AD^2 - 3AB^2)$.

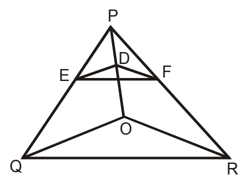
Q.39 In an isosceles ΔABC, AB = AC and BD $\perp$ AC. Prove that $(BD^2 - CD^2) = 2CD \cdot AD$

Q.40 In an isoscels ΔABC, AB = AC and D is a point on BC. Prove that $(AB^2 - AD^2) = BD \cdot CD$

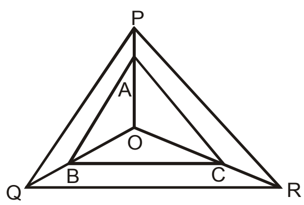
Theorem 6.1

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Q.41 In figure, DE || OQ and DF || OR. Show that EF||QR.



Q.42 In figure A,B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC||QR



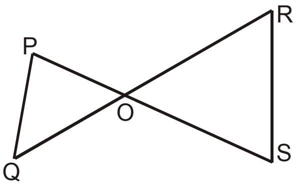
Q.43 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Q.44 The diagonals of quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Theorem 6.4

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

Q.45 In figure, if PQ || RS, prove that Δ POQ ~ Δ SOR.

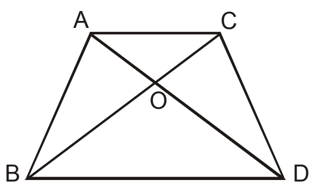


Q.46 A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Theorem 6.6

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Q.47 In figure ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$



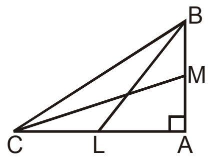
Q.48 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Theorem 6.8

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Q.49 BL and CM are medians of a triangle ABC right angled at A.

Prove that: $4(BL^2 + CM^2) = 5\ BC^2$.



Q.50 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Q.51 In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC. Prove that $9\ AD^2 = 7\ AB^2$.

ANSWER KEY

2. 2 **6.** 7 **7.** 40°

13. 60m **14.** 9 m **15.** 96 cm^2

16. 4.9 cm **19.** $\frac{1}{4}$ **20.** $2\sqrt{5}$

23. 120 cm^2 **27.** (i) 3 : 7 (ii) 14 cm

30. (i) $\frac{ab}{\sqrt{a^2+b^2}}$ **32.** (i) $\frac{1}{4}$ (ii) $\frac{1}{25}$ (iii) $\frac{1}{24}$

33. (i) 5 : 9 (ii) 5 : 14 (iii) 5 : 14

46. 1.6 m

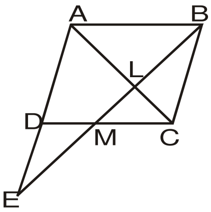
EXERCISE – II BOARD PROBLEMS

Q.1 P and Q are points on the sides CA and CB respectively of a ΔABC right-angled at C. Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$.

Q.2 ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.

Q.3 In ΔABC, if AD is the median, show that $AB^2 + AC^2 = 2\,[AD^2 + BD^2]$.

Q.4 In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM, when joined meets AC in L and AD produced in E. Prove that EL = 2BL.



Q.5 ABC is a right triangle, right-angled at C. If p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) pc = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Q.6 In an equilateral triangle PQR, the side QR is trisected at S. Prove that $9PS^2 = 7PQ^2$.

Q.7 If the diagonals of a quadrilateral divide each other proportionally, prove that it is trapezium.

Q.8 In an isosceles triangle ABC with AB = AC, BD is a perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD \cdot AD$.

Q.9 ABC and DBC are two triangles on the same base BC. If AD intersect BC at O. Prove that

$$\frac{ar.\,\Delta ABC}{ar.\,\Delta DBC} = \frac{AO}{DO}$$

Q.10 In ΔABC, $\angle A$ is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that AB × AE = AC × AD.

Q.11 Points P and Q are on sides AB and AC of a triangle ABC in such a way that PQ is parallel to side BC. Prove that the median AD drawn from vertex A to side BC bisects the segment PQ.

Q.12 If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

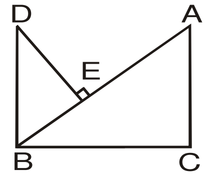
OR

Two Δs' ABC and DBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90°$. If CA and BD meet each other at E, show that AE.EC = BE.ED.

Q.13 D and E are points on the sides CA and CB respectively of ΔABC right-angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

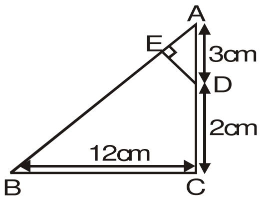
OR

In fig. DB ⊥ BC, DE ⊥ AB and AC ⊥ BC. Prove that



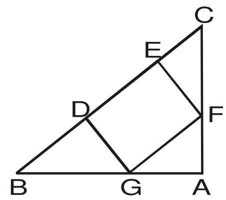
Q.14 E is a point on the side AD produced of a $||^{gm}$ ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$. **Foreign-2008**

Q.15 In fig, ΔABC is right angled at C and DE ⊥ AB. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.

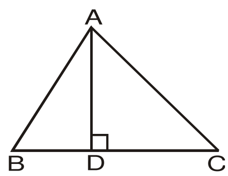


OR

In fig, DEFG is a square and $\angle BAC = 90°$. Show that $DE^2 = BD \times EC$

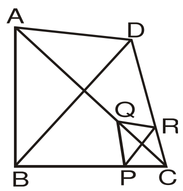


Q.16 In fig, AD ⊥ BC and BD = $\frac{1}{3}$ CD. Prove that $2CA^2 = 2AB^2 + BC^2$.



Q.17 In fig, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ||BA and PR||BD. Prove that QR||AD.

OR



Q.18 In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that

(i) $9AQ^2 = 9AC^2 + 4BC^2$

(ii) $9BP^2 = 9BC^2 + 4AC^2$

(iii) $9(AQ^2 + BP^2) = 13AB^2$.

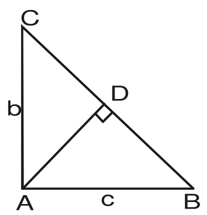
Q.19 The ratio of the areas of similar triangles is equal to the ratio of the squares on the corresponding sides, prove. Using the above theorem, prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

Q.20 Perpendiculars OD, OE and OF are drawn to sides BC, CA and AB respectively from a point O in the interior of a ΔABC. Prove that :

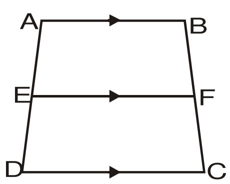
(i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$.

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

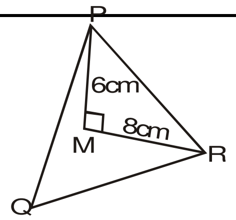
Q.21 In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides, prove. Using the above theorem, determine the length of AD in terms of b and C.



Q.22 If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove. Use this result to prove the following : In the given figure, if ABCD is a trapezium in which AB $\parallel$ DC $\parallel$ EF, then $\frac{AE}{ED} = \frac{BF}{FC}$.



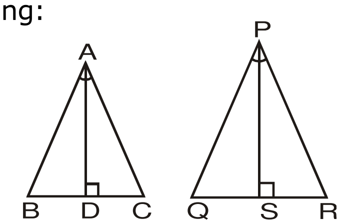
Q.23 State and prove pythagoras theorem. Use the theorem and calculate area (ΔPMR) from the given figure.



Q.24 In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the two sides. Given that $\angle$B of ΔABC is an acute angle and AD $\perp$ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.

Q.25 In a right triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using above, solve the following : In quadrilateral ABCD, find the length of CA, if CD $\perp$ DB, AB $\perp$ DB, CD = 6 m, DB = 12 m and AB = 11 m.

Q.26 Prove that the ratio of the areas of two similar triangles is equal to the squares of their corresponding sides. Using the above, do the following:



In fig. ΔABC and ΔPQR are isosceles triangles in which $\angle A = \angle P$. If $\frac{\text{area }(\Delta ABC)}{\text{area }(\Delta PQR)} = \frac{9}{16}$, find $\frac{AD}{PS}$.

Q.27 In a right-angled triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using the above result, find the length of the second diagonal of a rhombus whose side is 5 cm and one of the diagonals is 6 cm.

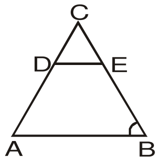
Q.28 In a triangle, if the square on one side is equal to the sum of the squares on the other two sides prove that the angle opposite the first side is a right angle.

Use the above theorem and prove thet following : In triangle ABC, AD $\perp$ BC and BD = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

Q.29 In a right triangle, prove that the square on hypotenuse is equal to sum of the squares on the other two sides. Using the above result, prove the following : PQR is a right triangle, right angled at Q. If S bisects QR, show that $PR^2 = 4\ PS^2 - 3\ PQ^2$.

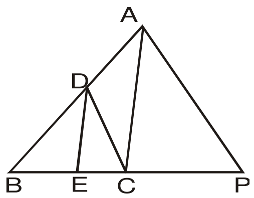
Q.30 If a line is drawn parallel to one side of a trial prove that the other two sides are divided in the same ratio. Using the above result, prove

from fig. that AD = BE if ∠A = ∠B and DE ∥ AB.



Q.31 Prove that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Apply the above theorem on the following : ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC.

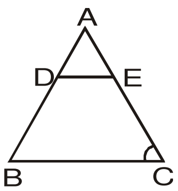
Q.32 If a line is drawn parallel to one side of a triangle, prove that the other two sides are divided in the same ratio. Use the above to prove the following : In the given figure DE ∥ AC and DC ∥ AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.



Q.33 In a triangle if the square on one side is equal to the sum of squares on the other two sides, prove that the angle opposite to the first side is a right angle. Use the above theorem to prove the following :

In a quadrilateral ABCD, ∠B = 90°. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that ∠ACD = 90°.

Q.34 If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In figure, DE ∥ BC and BD = CE. Prove that ABC is an isosceles triangle.



Q.35 Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Use the above for the following : If the areas of two similar triangles are equal, prove that they are congruent.

Q.36 Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Using the above result, prove the following :

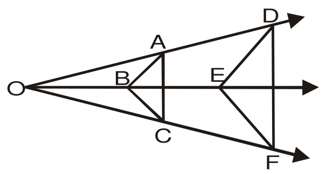
In a ΔABC, XY is parallel to BC and it divides ΔABC into two parts of equal area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

Q.37 Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides. Using the above, do the following:

The diagonals of a trapezium ABCD, with AB∥DC, intersect each other at the point O. If AB = 2 CD, find the ratio of the area of ΔAOB to the area of ΔCOD.

Q.38 If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following :

In the fig, AB∥DE and BC∥EF. Prove that AC∥DF.



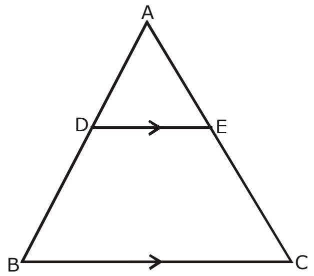
Q.39 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Using the above, do the following : In a trapezium ABCD, AC and BD are intersecting at O, AB∥DC and AB = 2 CD. If area of ΔAOB = 84 cm^2, find the area of ΔCOD.

ANSWER KEY

2.	$2\sqrt{5}$ cm	**15.**	AE = $\frac{15}{13}$, DE = $\frac{36}{13}$
21.	$\frac{bc}{\sqrt{b^2+c^2}}$	**23.**	24 cm^2
25.	13 cm	**26.**	3 : 4
27.	8 cm	**37.**	4 : 1
39.	21 cm^2		

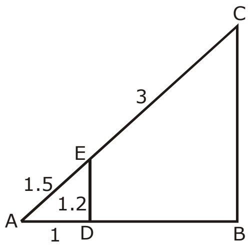
EXERCISE – III | NTSE /OLYMPIAD /FOUNDATION PROBLEMS

Q.1 In ΔABC, DE || BC, AD = 2.4cm, AE = 3.2 cm, and EC = 4.8 cm. The length of AB is:



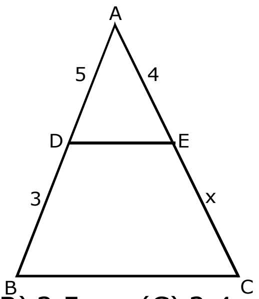
(A) 3.6 cm (B) 6cm
(C) 6.4cm (D) 1.6cm

Q.2 If $\Delta ADE \sim \Delta ABC$, then BC = ?



(A) 4.5 (B) 3 (C) 3.6 (D) 2.4

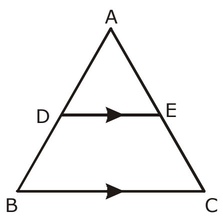
Q.3 In the given figure ED || BC. The value of x is:



(A) 2.8 (B) 2.5 (C) 2.4 (D) 4

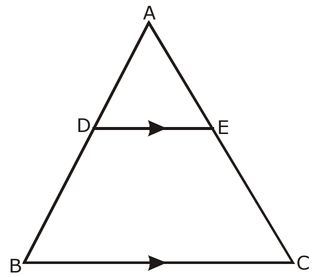
Q.4 The line segments joining the midpoints of the sides of a triangle form four triangles, each of which is
(A) congruent to the original triangle
(B) similar to the original triangle
(C) an isosceles triangle
(D) an equilateral triangle

Q.5 In the ΔABC DE || BC, AD = 1.7 cm, AB = 6.8 cm and AC = 9 cm. The length of AE is



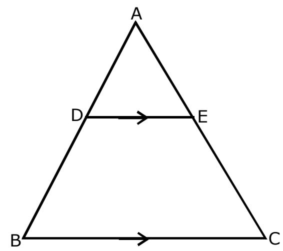
(A) 2.5 cm (B) 4.5 cm
(C) 2.2 cm (D) 7.3 cm

Q.6 In the ΔABC, DE || BC, AD = (7x − 4) cm, AE = (5x − 2) cm, DB = (3x + 4) cm, and EC = 3x cm. The value of x is: ?



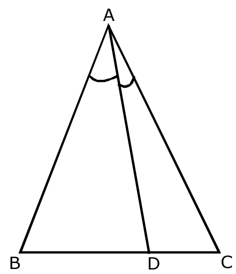
(A) 3 (B) 5 (C) 4 (D) 2.5

Q.7 In the ΔABC, DE || BC and $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6 cm, then the length of AE is :



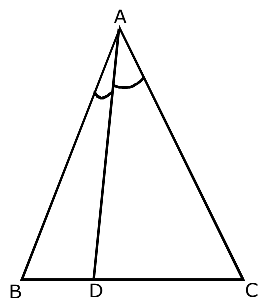
(A) 4.2 cm (B) 3.1cm
(C) 2.1 cm (D) 2.8 cm

Q.8 In ΔABC, AD is the internal bisector of $\angle A$. If BD = 5 cm, BC= 7.5 cm, then AB : AC is equal to :



(A) 2 : 1 (B) 1 : 2 (C) 4 : 5 (D) 3 : 5

Q.9 In the ΔABC, AD is the internal bisector of $\angle A$. If BD = 4 cm, DC= 5 cm and AB = 6 cm, then the length of AC = ?



(A) 3 cm (B) 8 cm
(C) 4.5 cm (D)7.5 cm

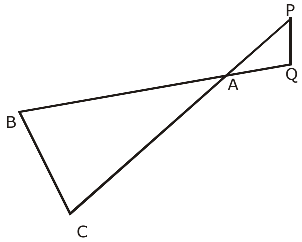
Q.10 In a right triangle PQR, $PR^2 + PQ^2 = QR^2$. Which angle is equal to 90°?

(A) $\angle P$ (B) $\angle Q$
(C) $\angle R$ (D) none of these

Q.11 The perimeter of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9cm, then the corresponding side of the second triangle is:

(A) 4.5 cm (B) 5cm
(C) 3.5cm (D) 5.4 cm

Q.12 In the given figure, $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, then the length of AQ is:

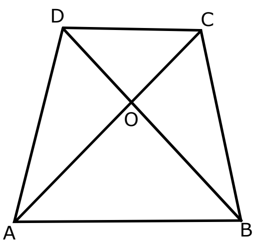


(A) 3.25 cm (B) 4cm
(C) 4.25cm (D) 3 cm

Q.13 The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, the longest side of the smaller triangle is

(A) 21 cm (B) 22cm
(C) 24 cm (D) 20 cm

Q.14 In the given figure, AB || DC and diagonals AC and BD intersect at O. If AO = (3x −1) cm, BO = (2x +1) cm, OC = (5x −3) cm and OD = (6x − 5) cm, then the value of x is :



(A) 2 (B) 3
(C) 2.5 (D) 4

Q.15 $\Delta ABC \sim \Delta DEF$ and the perimeters of ΔABC and ΔDEF are 30 cm and 18 cm respectively. If BC = 9 cm, then EF is equal to:

(A) 6.3 (B) 5.4
(C) 7.2 (D) 4.5

Q.16 $\Delta ABC \sim \Delta DEF$ such that AB = 9.1 cm and DE = 6.5 cm. If the perimeter of ΔDEF is 25 cm, then the perimeter of ΔABC is :

(A) 35 cm (B) 28 cm
(C) 42 cm (D) 40 cm

Q.17 If D is a point on the side AB of ΔABC such that AD : DB = 3 : 2 and E is a point on BC such that DE || AC. The ratio of areas of ΔABC and ΔBDE is

(A) 4 : 25 (B) 25 : 4
(C) 5 : 4 (D) 4 : 5

Q.18 In an equilateral triangle ABC, if $AD \perp BC$, then:

(A) $2AB^2 = 3AD^2$ (B) $4AB^2 = 3AD^2$
(C) $3AB^2 = 4AD^2$ (D) $3AB^2 = 2AD^2$

Q.19 The line segments joining the mid points of the adjacent sides of a quadrilateral form a:

(A) paralle logram (B) square
(C) rhombus (D) rectangle

Q.20 If the diagonals of a quadrilateral divide each other proportionally, then it is a:

(A) paralle logram (B) trapezium
(C) rectangle (D) square

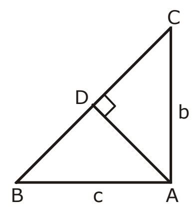
Q.21 A right triangle has hypotenuse of length p cm and one side of length q cm. If p − q = 1, then the length of the third side of the triangle is :

(A) $\sqrt{2q+1}$ (B) $\sqrt{2p+1}$
(C) 2p (D) 1 + q

Q.22 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, then ΔABC is right angled at:

(A) $\angle A$ (B) $\angle B$
(C) $\angle C$ (D) none of these

Q.23 In the figure, ΔABC is a right triangle, right angled at A and $AD \perp BC$. If AB = c and AC = b, then AD is equal to:



(A) $\dfrac{bc}{\sqrt{b^2+c^2}}$ (B) $\dfrac{bc}{b^2+c^2}$

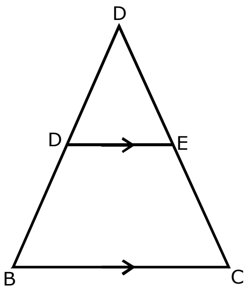
(C) $\dfrac{b^2c}{\sqrt{b^2+c^2}}$ (D) none of these

Q.24 A triangle has sides 5cm, 12 cm and 13 cm. The length of the perpendicular from the opposite vertex to the side whose length is 13 cm is:

(A) 4.9 (B) 3.6 cm

(C) 5.5 cm (D) 4.6 cm

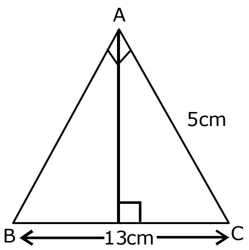
Q.25 In the given figure, DE || BC. If DE= 3 cm, BC = 6 cm and ar (ΔADE) = 15 cm^2. Area of ΔABC is:



(A) 30 cm^2 (B) 60 cm^2

(C) 40 cm^2 (D) 50 cm^2

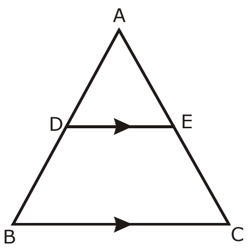
Q.26 ΔABC is right angled at A and AD $\perp$ BC. If BC = 13 cm and AC = 5 cm, the ratio of the areas of ΔABC and ΔADC is :



(A) 25 : 169 (B) 169 : 25

(C) 5 : 13 (D) 13 : 5

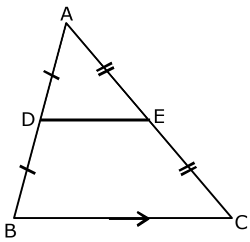
Q.27 In the given figure, DE || BC and DE : BC = 3 : 5. The ratio of the areas of ΔADE and the trapezium BCED is :



(A) 9 : 16 (B) 16 : 9

(C) 3 : 5 (D) 5 : 3

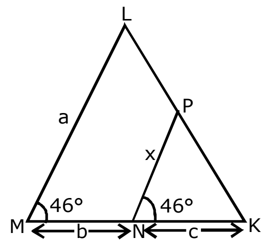
Q.28 In ΔABC, D and E are the mid points of AB and AC respectively. The ratio of the areas of ΔADE and ΔABC is :



(A) 1 : 2 (B) 2 : 1
(C) 1 : 4 (D) 4 : 1

Q.29 In a trapezium ABCD, O is the point of intersection of AC and BD, AB || CD and AB = 2 × CD. If the area of ΔAOB = 84 cm^2, then the area of ΔCOD is :
(A) 25 cm^2 (B) 21 cm^2
(C) 24 cm^2 (D) 32 cm^2

Q.30 In the given figure, x in terms of a,b and c is :



(A) $x = \frac{ac}{b+c}$ (B) $x = \frac{ab}{b+c}$

(C) $x = \frac{ac}{a+b}$ (D) $x = \frac{bc}{a+c}$

Q.31 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, the distance between their tops is
(A) 12 m (B) 13 m
(C) 15 m (D) 11 m

Q.32 Two isosceles triangles have their corresponding angles equal and their areas are in the ratio 25 : 36. The ratio of their corresponding height is
(A) 25 : 36 (B) 36 : 25
(C) 5 : 6 (D) 6 : 5

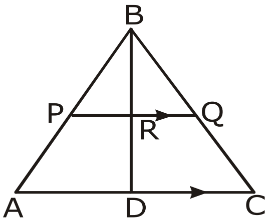
Q.33 A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time , a tower casts a shadow 50 m long on the ground. The height of the tower is:
(A) 100 m (B) 120 m
(C) 25 m (D) 200 m

Q.34 The length of altitude AD of an isosceles ΔABC, in which AB = AC = 2a units and BC = a units, is :

(A) $\frac{a\sqrt{15}}{4}$ (B) $\frac{a\sqrt{15}}{2}$

(C) $\frac{\sqrt{15a}}{4}$ (D) none of these

Q.35 In the given figure, if BD is the bisector of $\angle B$, PQ || AC, then :



(A) PR × QR = BQ × BP
(B) PR × BQ = QR × BP
(C) both (A) and (B)
(D) None of these

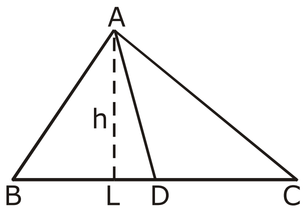
Q.36 $\Delta ABC \sim \Delta DEF$ such that ar(ΔABC) = 36 cm^2 and ar(ΔDEF) = 49 cm^2. Then, the ratio of their corresponding sides is

(A) 36 : 49 (B) 6 : 7

(C) 7 : 6 (D) $\sqrt{6} : \sqrt{7}$

Q.37 In ΔABC, if AD is the bisector of $\angle A$, then

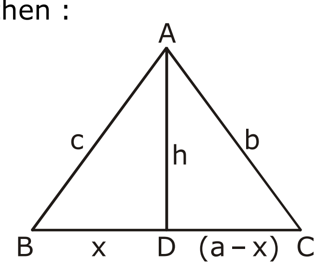
$\frac{ar\ (\Delta ABD)}{ar\ (\Delta ADC)}$ is equal to



(A) $\frac{BD}{DC}$ (B) $\frac{AB}{AC}$

(C) both A and B (D) none of these

Q.38 In the given figure, $\angle B < 90^o$ and segment AD $\perp$ BC, then :



(A) $b^2 = h^2 + a^2 + x^2 - 2ax$
(B) $b^2 = h^2 - a^2 - x^2 + 2ax$
(C) $b^2 = h^2 + a^2$
(D) none of these

Q.39 If ABC is an isosceles triangle and D is a point on BC such that AD $\perp$ BC, then :
(A) $AB^2 - AD^2 = BD \cdot DC$
(B) $AB^2 - AD^2 = BD^2 - DC^2$
(C) $AB^2 + AD^2 = BD \cdot DC$
(D) $AB^2 + AD^2 = BD^2 \cdot DC^2$

Q.40 ΔABC is a right angle right-angled at A and AD $\perp$ BC. Then $\frac{BD}{DC}$ is equal to :

(A) $\left(\frac{AB}{DC}\right)^2$ (B) $\frac{AB}{AC}$

(C) $\left(\frac{AB}{AD}\right)^2$ (D) $\frac{AB}{AD}$

Q.41 If each side of a rhombus is 10 cm and one of its diagonals is 16 cm, then the length of the other diagonal is :

(A) 16 cm (B) 14 cm

(C) 12 cm (D) 10 cm

Q.42 D, E and F are the mid-points of side AB, BC and CA respectively of ΔABC. The ratio of the areas of ΔDEF and ΔABC is :

(A) 1 : 2 (B) 4 : 1

(C) 3 : 4 (D) 1 : 4

Q.43 An aeroplane leaves an airport and flies due north at a speed of 1000 km/h. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/h. After $1\frac{1}{2}$ hours, the distance between the two planes is :

(A) $300\sqrt{61}$ km (B) $61\sqrt{300}$ km

(C) $\sqrt{36100}$ km (D) $30\sqrt{61}$ km

Q.44 ABC is a right triangle, right angled at C. If p is the length of the perpendicular from C to AB, AB = c, BC = a and AC = b, then :

(A) $\frac{1}{a^2} = \frac{1}{b^2} - \frac{1}{p^2}$ (B) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(C) $\frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{a^2}$ (D) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Q.45 A vertical pole 5m long casts a shadow 2m long on the ground. At the same time, a tower casts a shadow 25m long on the ground. The height of the tower is

(A) 12.5 m (B) 50 m

(C) 100 m (D) 75 m

Q.46 A vertical stick 1.2m long casts a shadow 40cm long on the ground. At the same time a pole 6m high casts a shadow x on the ground. the value of x is

(A) 3m (B) 12m

(C) 2m (D) 18m

Q.47 Two poles 6m and 11m high stand vertically on the ground. If the distance between their feet is 12m, then the distance between their tops is

(A) 11m (B) 14m

(C) 12m (D) 13m

Q.48 ΔABC and ΔDEF, are two similar triangles such that $\angle A = 36^o$ and $\angle E = 74^o$, then $\angle C$ is

(A) 70^o (B) 50^o

(C) 60^o (D) 80^o

Q.49 Corresponding sides of two similar triangles are in the ratio of 5 : 7. Areas of these triangles are in the ratio of

(A) $\sqrt{5} : \sqrt{7}$ (B) 7 : 5

(C) 25 : 49 (D) 49 : 25

Q.50 The area of two similar triangles are 25 sq cm and 121 sq cm. The ratio of their corresponding sides is

(A) 5 : 11 (B) 11 : 5

(C) $\sqrt{5} : \sqrt{11}$ (D) $\sqrt{11} : \sqrt{5}$

Q.51 In ΔABC, AB = 2cm, BC = 3cm and AC = 2.5cm If $\Delta DEF \sim \Delta ABC$ and EF = 6cm, then perimeter of ΔDEF is

(A) 7.5 cm (B) 15cm

(C) 22.5cm (D) 30cm

Q.52 In ΔABC, and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{3}{4}$, then area ($\Delta$ABC) : Area ($\Delta$DEF) is equal to

(A) 3 : 4 (B) 16 : 9

(C) 9 : 16 (D) 27 : 64

Q.53 DE is drawn parallel to base BC of ΔABC meeting AB at D and AC at E. If $\frac{AB}{BD} = 4$ and CE = 2cm, then AE is equal to

(A) 2cm (B) 4cm

(C) 6cm (D) 8cm

Q.54 In ΔPQR, G and H are points on PQ and PR respecively such that GH || QR and PG : GQ = 3 : 1. If PH = 3.3 cm then PR is equal to

(A) 1.1 cm (B) 4cm

(C) 5.5 cm (D) 4.4 cm

Q.55 ΔABC and ΔBDE are two equilateral triangles such that D is mid-point of BC. The ratio of areas of triangles ABC and BDE is

(A) 4 : 1 (B) 2 : 1

(C) 1 : 2 (D) 1 : 4

Q.56 In a ΔABC, D andE are points on sides AB and AC respectivley such that BCED is a trapezium. If $\frac{DE}{BC} = \frac{3}{5}$, then $\frac{\text{Area }(\Delta ADE)}{\text{Area (Trap.BCED)}}$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$

(C) $\frac{3}{5}$ (D) $\frac{9}{25}$

Q.57 Two isosceles triangles have equal angles and their areas are as 16 : 25. The ratio of their corresponding heights is

(A) 3 : 2 (B) 5 : 4

(C) 5 : 7 (D) 4 : 5

Q.58 ABCD is a trapezium in which BC || AD. If AB = 4cm and the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC is equal to

(A) 7 cm (B) 8 cm

(C) 6 cm (D) 9 cm

Q.59 In a ΔABC, $\angle A = 90^o$, AB = 5cm and AC = 12cm. If $AD \perp BC$, then AD is equal to

(A) $\frac{13}{2}$ cm (B) $\frac{13}{60}$ cm

(C) $\frac{60}{13}$ cm (D) $\frac{2\sqrt{15}}{13}$ cm

Q.60 A man goes 24 m due west and then 10 m due north. How far is the from the starting point?

(A) 34 m (B) 17 m

(C) 26 m (D) 28 m

Q.61 A man goes 12 m due south and then 35 m due west. How far is he from the starting point?

(A) 47 m (B) 23.5 m

(C) 23 m (D) 37 m

Q.62 Two poles of height 13 m and 7 m respectively stand vertically on a plane ground at a distance of 8 m from each other. The distance between their tops is

(A) 9 m (B) 10 m

(C) 11 m (D) 12 m

Q.63 A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high?

(A) 2.4 m (B) 1.35 m

(C) 1.5 m (D)13.5 m

Q.64 A vertical pole 6 m long casts a shadow of length 3.6 m on the ground. What is the height of a tower which casts a shadow of length 18m at the same time?

(A) 10.8 m (B) 28.8 m

(C) 32.4 m (D) 30 m

Q.65 A ladder 25 m long just reaches the top of building 24 m high from the ground. What is the distance of the foot of the ladder from the building?

(A) 7 m (B) 14 m

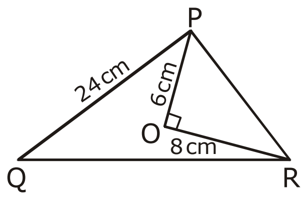
(C) 21 m (D)24.5 m

Q.66 A ladder 15 m long reaches a window which 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. The width of the street is

(A) 27 m (B) 21 m

(C) 24 m (D) 18 m

Q.67 In the given figure, O is a point inside a ΔPQR such that $\angle PQR = 90^o$. OP = 6 cm and OR = 8 cm. If PQ = 24 cm and $\angle QPR = 90^o$, then QR = ?



(A) 28 cm (B) 25 cm

(C) 26 cm (D) 32 cm

Q.68 The hypotenuse of a right triangle is 25 cm. The other two sides are such that one is 5 cm longer than the other. The length of these sides are

(A) 10 cm, 15 cm (B) 15 cm, 20 cm

(C) 12 cm, 17 cm (D) 13 cm, 18 cm

Q.69 The height of an equilateral triangle having each side 12 cm, is

(A) $6\sqrt{2}$ cm (B) $6\sqrt{3}$ cm

(C) $3\sqrt{6}$ cm (D) $6\sqrt{6}$ cm

Q.70 ΔABC is an isosceles triangle with AB = AC = 13 cm and the length of altitude from A on BC is 5 cm. Then, BC = ?

(A) 12 cm (B) 16 cm

(C) 18 cm (D) 24 cm

Q.71 The measures of three angles of a triangle are in the ratio 1 : 2 : 3. Then, the triangle is

(A) right-angled (B) equilateral

(C) isosceles (D) obtuse-angled

Q.72 For a ΔABC, which is true?

(A) $AB - AC = BC$ (B) $(AB - AC) > BC$

(C) $(AB - AC) < BC$ (D) None of these

Q.73 In a triangle, the perpendicular from the vertex to the base bisect the base. The triangle is

(A) right-angled (B) isosceles

(C) scalene (D) obtuse-angled

Q.74 In a rhombus of side 10 cm, one of the diagonals is 12 cm long. The length of the second diagonal is

(A) 20 cm (B) 18 cm

(C) 16 cm (D) 22 cm

Q.75 The length of the diagonals of a rhombus are 24 cm and 10 cm. The length of each side of the rhombus is

(A) 12 cm (B) 13 cm

(C) 14 cm (D) 17 cm

Q.76 If the diagonals of a quadrilateral divide each other proportionally, then it is a

(A) parallelogram (B) trapezium

(C) rectangle (D) square

Q.77 The line segment joining the midpoints of the adjacent sides of a quadrilateral form

(A) a parallelogram (B) a rectangle

(C) a square (D) rhombus

Q.78 If the bisector of an angle of a triangle bisects the opposite side, then the triangle is

(A) scalene (B) equilateral

(C) isosceles (D) right-angled

Q.79 In ΔABC, it is given that $AB = 9$ cm, $BC = 6$ cm and $CA = 7.5$ cm. Also, ΔDEF is given such that $EF = 8$ cm and $\Delta DEF \sim \Delta ABC$. Then, perimeter of ΔDEF is

(A) 22.5 cm (B) 25 cm

(C) 27 cm (D) 30 cm

Q.80 It is given that $\Delta ABC \sim \Delta DEF$. If $\angle A = 30^o$. $\angle C = 50^o$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm, then which of the following is ture?

(A) $DE = 12$ cm, $\angle F = 50^o$

(B) $DE = 12$ cm, $\angle F = 100^o$

(C) $EF = 12$ cm, $\angle D = 100^o$

(D) $EF = 12$ cm, $\angle D = 30^o$

Q.81 In ΔABC and ΔDEF, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$, then

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$

(C) $\angle B = \angle D$ (D) $\angle A = \angle F$

Q.82 In $\angle DEF$ and ΔPQR, it is given that $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not ture?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QE}$

Q.83 If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(A) $BC \cdot EF = AC \cdot FD$ (B) $AB \cdot EF = AC \cdot DE$

(C) $BC \cdot DE = AB \cdot EF$ (D) $BC \cdot DE = AB \cdot FD$

Q.84 In ΔABC and ΔDEF, it is given that ∠B = ∠E, ∠F = ∠C and AB = 3DE, then the two triangles are

(A) congruent but not similar

(B) similar but not congrunet

(C) neither congruent nor similar

(D) similar as well as congruent

Q.85 If in ΔABC and ΔPQR, we have: $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(A) ΔPQR ~ ΔCAB (B) ΔPQR ~ ΔABC

(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

Q.86 It is given that ΔABC ~ ΔDEF and the corresponding sides of these triangles are in the ratio 3 : 5. Then ar(ΔABC) : ar(ΔDEF) =

(A) 3 : 5 (B) 5 : 3

(C) 9 : 25 (D) 25 : 9

Q.87 It is given that ΔABC ~ ΔPQR and $\frac{BC}{QR} = \frac{2}{3}$, then $\frac{ar(\Delta PQR)}{ar(\Delta ABC)}$ = ?

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{4}{9}$ (D) $\frac{9}{4}$

Q.88 In ΔABC and ΔDEF, we have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then (ΔABC) : ar(ΔDEF) = ?

(A) 5 : 7 (B) 25 : 49

(C) 49 : 25 (D) 125 : 343

Q.89 In right ΔABC, BC = 7 cm, AC – AB = 1 cm and ∠B = 90°. The value of cosA + cosB + cosC is

(A) $\frac{1}{7}$ (B) $\frac{32}{24}$

(C) $\frac{31}{25}$ (D) $\frac{25}{31}$

Q.90 Given that ΔABC ~ ΔPQR and

$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{16}{9}$.

If PQ = 18 cm and BC = 12 cm. Then, AB and QR are, respectively

(A) 9 cm, 24 cm

(B) 24 cm, 9 cm

(C) 32 cm, 6.75 cm

(D) 13.5 cm, 16 cm

Q.91 ΔABC is an equilateral triangle of side $2\sqrt{3}$ cm, O is any point in the interior of ΔABC. If x, y, z are the distances of O from the sides of the triangle, then x + y + z is equal to

(A) 2 + $\sqrt{3}$ cm (B) 3 cm

(C) 4 cm (D) 5 cm

ANSWER KEY

1.	B	**2.**	C	**3.**	C	**4.**	B
5.	C	**6.**	C	**7.**	C	**8.**	A
9.	D	**10.**	A	**11.**	D	**12.**	A
13.	B	**14.**	A	**15.**	B	**16.**	A
17.	B	**18.**	C	**19.**	A	**20.**	B
21.	A	**22.**	C	**23.**	A	**24.**	D
25.	B	**26.**	B	**27.**	A	**28.**	C
29.	B	**30.**	A	**31.**	B	**32.**	C
33.	A	**34.**	B	**35.**	B	**36.**	B
37.	C	**38.**	A	**39.**	A	**40.**	B
41.	C	**42.**	D	**43.**	A	**44.**	D
45.	B	**46.**	C	**47.**	D	**48.**	A
49.	C	**50.**	A	**51.**	B	**52.**	C
53.	C	**54.**	D	**55.**	A	**56.**	B
57.	D	**58.**	B	**59.**	C	**60.**	C
61.	D	**62.**	B	**63.**	C	**64.**	D
65.	A	**66.**	B	**67.**	C	**68.**	B
69.	B	**70.**	D	**71.**	A	**72.**	C
73.	B	**74.**	C	**75.**	B	**76.**	B
77.	A	**78.**	C	**79.**	D	**80.**	B
81.	C	**82.**	B	**83.**	C	**84.**	B
85.	A	**86.**	C	**87.**	D	**88.**	B
89.	C	**90.**	B	**91.**	B		

COORDINATE GEOMETRY

INTRODUCTION

In the previous class, you have learnt to locate the position of a point in a coordinate plane of two dimensions, in terms of two coordinates. You have learnt that a linear equation in two variables, of the form $ax + by + c = 0$ (either $a \neq 0$ or $b \neq 0$) can be represented graphically as a straight line in the coordinate plane of x and y coordinates. In chapter 4, you have learnt that graph of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is an upward parabola if $a > 0$ and a downward parabola if $a < 0$.

In this chapter, you will learn to find the distance between two given points in terms of their coordinates and also, the coordinates of the point which divides the line segment joining the two given points internally in the given ratio.

HISTORICAL FACTS

Rene Descartes (1596-1650), the 17th century French-Mathematician, was a thinker and a philosopher. He is called the father of Co-ordinate Geometry because he unified Algebra and Geometry which were earlier two distinct branches of Mathematics. Descartes explained that two numbers called co-ordinates are used to locate the position of a point in a plane.

He was the first Mathematician who unified Algebra and Geometry, so Analytical Geometry is also called Algebraic Geometry. Cartesian plane and Cartesian Product of sets have been named after the great Mathematician.

REVIEW

1. Rectangular co-ordinate system –

(a) Distance between two points; The distance between the points

P (x_1, y_1) and Q (x_2, y_2) is given by $PQ = \sqrt{(x_2 - x_1)^2 + y_2 - y_1)^2}$

(b) Section formula

(i) If P (x, y) divides the join of A (x_1, y_1) and B (x_2, y_2) internally in the ratio m : n then

$$x = \left(\frac{mx_2 + nx_1}{m+n}\right), y = \left(\frac{my_2 + ny_1}{m+n}\right)$$

(ii) If If P (x, y) divides the join of A (x_1, y_1) and B (x_2, y_2) externally in the ratio m : n then

$$x = \left(\frac{mx_2 - nx_1}{m-n}\right), y = \left(\frac{my_2 - ny_1}{m-n}\right)$$

(iii) If A (x_1, y_1) and B (x_2, y_2) be two given points then the co-ordinate of p mid point of AB are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

<u>SUMMARY OF THE CHAPTER</u>

BASIC CONCEPTS AND IMPORTANT RESULTS

* **Coordinate system**

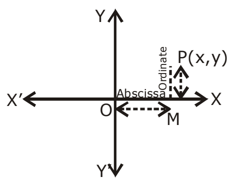
When two numbered lines perpendicular to each other (usually horizontal and vertical) are placed together so that the two origins (the points corresponding to zero) coincide the resulting configuration is called a **cartesian coordinate system** or simply a **coordinate system** or a **coordinate plane.**

Let X'OX and Y'OY, two number lines perpendicular to each other, meet at the point O (shown in the adjoining figure), then

(i) X'OX is called x-axis.
(ii) Y'OY is called y-axis.
(iii) X'OX and Y'OY taken together are called coordinate axes
(iv) the point O is called the **origin.**

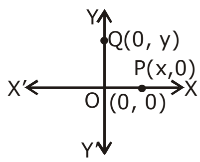
* **Coordinates of a point**

Let P be any point in the coordinate plane. From P, draw PM perpendicular to X'OX, then
(i) OM is called x-coordinate or abscissa of P and is usually denoted by x.
(ii) MP is called y-coordinate or ordinate of P and is usually denoted by
(iii) x and y taken together are called cartesian coordinates or simply coordinates of P and are written as by (x, y)



REMARKS

1. The coordinates of the origin O are (0, 0).
2. For any point on x-axis, its ordinate is always zero and so the coordinate of any ponit P on x-axis are (x, 0).
3. For any point on y-axis, its abscissa is always zero and so the coordinates of any point Q on y-axis are (0, y)



* **Coordinate Geometry**

Coordinate geometry is that branch of mathematics which deals with the study of geometry by mean of algebra. In coordinate geometry, we represent a point in a plane by an ordered pair of real numbers called coordinates of the point; and a straight line or a curve by an algebraic equation with real coefficients. We have seen earlier that a linear equation in two variables of the from $ax + by + c = 0$ (a, b not simultaneously zero) represents a straight line and the equation $y = ax^2 + bx + c$ $(a \neq 0)$ represents a parabola (upwards or downwards). In fact, coordinate geometry has been developed as an algebraic tool for studying geometry of figures. Thus, we use algebra advantageously to the study of straight lines and geometric curves.

* **Distance formula**

The distance between the points P (x_1, y_1) and Q(x_2, y_2) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The distance of the point P(x, y) from the origin O(0, 0) is given by OP = $\sqrt{x^2 + y^2}$.

* **Section formula**

The coordinates of the point which divides (internally) the line segment joining the points P(x_1, y_1) and Q(x_2, y_2) in the ratio

$m_1 : m_2$ are $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

* **Mid-point formula**

The coordinates of the mid-point of the line segment joining the points P (x_1, y_1) and Q(x_2, y_2) are

$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

REMARK. In problems where it is required to find the ratio when a given point divides the join of two given points, it is convenient to take the ratio as k : 1, for, in this way two unknowns (m_1 and m_2) are reduced to one unknown and the section formula becomes

$x = \frac{kx_2 + x_1}{k+1}$ and $y = \frac{ky_2 + y_1}{k+1}$.

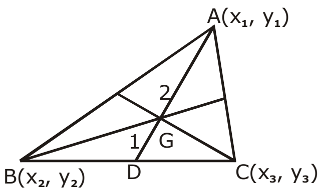
Then equate the abscissa or the ordinate of the point so obtained with that of the given point, and find the value of unknown k.

* **Centroid of a triangle**

The point where the medians of a triangle meet is called the ***centroid of the triangle.***

If AD is a medina of the triangle ABC and G is its centroid, then $\frac{AG}{GD} = \frac{2}{1}$. The coordinates of the point G are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$



REMARK. To prove that a quadrilateral is a

(i) ***Parallelogram*** **:** Show that opposite side are equal

Or

Show that diagonals bisect each other.

(ii) ***Rectangle*** **:** Show that opposite sides are equal and diagonals are also equal

Or

Show that opposite sides are equal and one angle is 90º

Or

Show that diagonals bisects each other and are equal.

(iii) ***Rhombus*** **:** Show that all sides are equal

Or

Show that diagonal bisect each other two adjacent sides are equal.

(iv) ***Square*** **:** Show that all sides are equal and diagonals are also equal

Or

Show that all sides are equal and one angle is 90º

Or

Show that diagonals bisect each other and two adjacent sides are equal and diagonals are also equal.

* **Area of a triangle**

The area of the triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is the absolute value (numerical value) of the expression

$$\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}.$$

* **Condition of collinearity of three points**

The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if and only if the area of $\Delta ABC = 0$

i.e., if and only if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0.$

SOLVED PROBLEMS

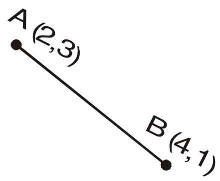
Ex.1 Find the distance between the following pairs of points : **[NCERT]**

(a) (2,3), (4, 1) (b) (–5, 7), (–1,3) (c) (a, b), (– a, – b)

Sol. (a) The given points are : A (2, 3), B (4, 1).

Required distance = AB = BA =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

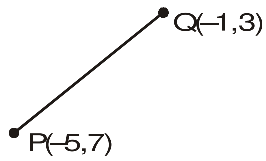


$$AB = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2}$$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(b) Distance between P (–5, 7) and Q (– 1, 3) is given by

$$PQ = QP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$= \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{16+16} = \sqrt{32}$$

Required distance = PQ = QP = $4\sqrt{2}$ units

(c) Distance LM between L (a,b) and M (–a, –b) is given by

$$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$= \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$ units

Ex.2 Find points on x-axis which are at a distance of 5 units from the point A(–1, 4).

Sol. Let the point on x-axis be P(x, 0).

Distance = PA = 5 units

$\Rightarrow$ $PA^2 = 25$ $\Rightarrow$ $(x+1)^2+(0-4)^2 = 25$

$\Rightarrow$ $x^2 + 2x + 1 + 16 = 25$ $\Rightarrow x^2 + 2x + 17 = 25$ $\Rightarrow$ $x^2 + 2x - 8 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 + 4(8)}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$

$$= \frac{-2+6}{2}, \frac{-2-6}{2} = \frac{4}{2}, -\frac{8}{2} = 2, -4$$

Required point on x-axis are (2, 0) and (–4, 0)

Verification : PA

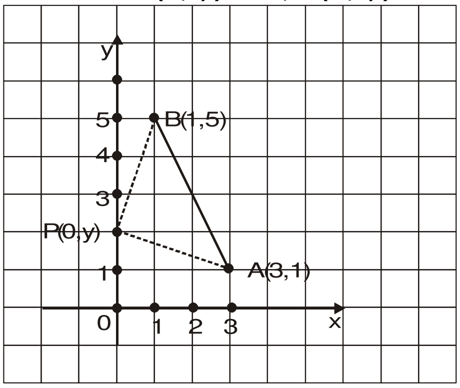
$$= \sqrt{(2+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$PA = \sqrt{(-4+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Ex.3 What point on y-axis is equidistant from the points (3, 1) and (1, 5) ?

Sol. Since the required point P(say) is on the y-axis, its abscissa (x-co-ordinate) will be zero. Let the ordinate (y-co-ordinate) of the point be y.

Therefore co-ordinates of the point P are : (0, y) i.e., P(0, y)



Let A and B denote the points (3, 1) and (1, 5) respectively.

PA = PB ...(given) Squaring we get :

$PA^2 = PB^2$

$\Rightarrow$ $(0 - 3)^2 + (y - 1)^2 = (0 - 1)^2 + (y - 5)^2$

$\Rightarrow$ $9 + y^2 + 1 - 2y = 1 + y^2 + 25 - 10y$

$\Rightarrow$ $y^2 - 2y + 10 = y^2 - 10y + 26$

$\Rightarrow$ $-2y + 10y = 26 - 10 \Rightarrow 8y = 16 \Rightarrow y = 2$

The required point on y-axis equidistant from A(3, 1) and B(1, 5) is P(0, 2).

Ex.4 If Q(2, 1) and R(–3, 2) and P(x, y) lies on the right bisector of QR then show that $5x - y + 4 = 0$.

Sol. Let P(x, y) be a point on the right bisector of QR : Q(2, 1) and R(–3, 2) are equidistant from P(x, y), then we must have :

PQ = PR

$\Rightarrow$ $PQ^2 = PR^2$

$\Rightarrow$ $(x - 2)^2 + (y - 1)^2 = (x + 3)^2 + (y - 2)^2$

$\Rightarrow$ $(x^2 - 4x + 4) + (y^2 - 2y + 1)$

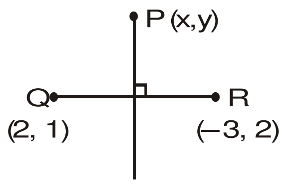
$= (x^2 + 6x + 9) + (y^2 - 4y + 4)$

$\Rightarrow$ $-4x - 2y + 5 = 6x - 4y + 13$

$\Rightarrow$ $10x - 2y + 8 = 0$

$\Rightarrow$ $2(5x - y + 4) = 0$

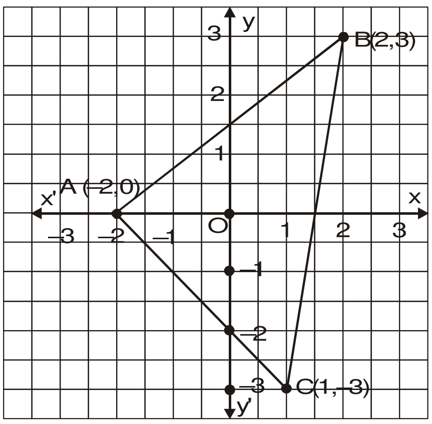
$\Rightarrow$ $5x - y + 4 = 0$



Ex.5 The vertices of a triangle are (–2, 0), (2, 3) and (1, –3). Is the triangle equilateral : isosceles or scalene?

Sol. We denote the given point (–2, 0), (2, 3) and (1, –3) by A, B and C respectively then :

A(–2, 0), B(2, 3), C(1, –3)



$AB = \sqrt{(2+2)^2 + (3-0)^2} = \sqrt{(4)^2 + (3)^2} = 5$

$BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{37}$

$CA = \sqrt{(-2-1)^2 + (0+3)^2} = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$

Thus we have $AB \neq BC \neq CA$

$\Rightarrow$ ABC is a scalene triangle

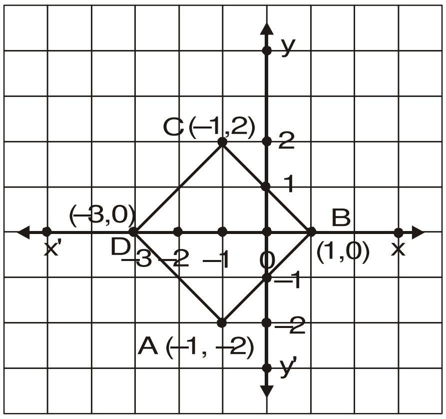
Ex.6 Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.
$(-1, -2), (1, 0), (-1, 2), (-3, 0)$ **[NCERT]**

Sol. $A(-1, -2), B(1, 0), C(-1, 2), D(-3, 0)$

Determine distances : AB, BC, CD, DA, AC and BD.

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$



$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

AB = BC = CD = DA

The sides of the quadrilateral are equal .... (1)

$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$$

$$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$$

The diagonals of quadrilateral are equal .... (2)

From (1) and (2) we conclude that ABCD is a square.

Ex.7 Determine whether the points (1, 5), (2, 3) and (–2, –11) are collinear. **[NCERT]**

Sol. The given points are : A(1, 5), B(2, 3) and C(–2, –11).

Let us calculate the distance : AB, BC and CA by using distance formula.

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$$

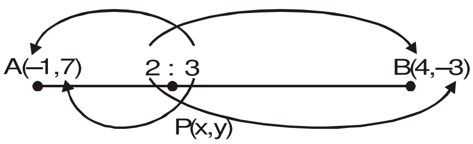
$$CA = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

From the above we see that : $AB + BC \neq CA$

Hence the above stated points A(1, 5), B(2, 3) and C(–2, –11) are not collinear.

Ex.8 Find the co-ordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

Sol. Let P(x, y) divides the line segment AB joining A(–1, 7) and B(4, –3) in the ratio 2 : 3. Then by using section formula the co-ordinates of P are given by : **[NCERT]**

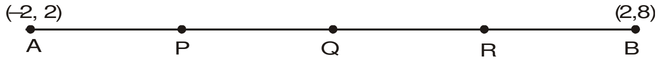


$$\left(\frac{2\times4\times3\times(-1)}{2+3}, \frac{2\times(-3)+3\times7}{2+3}\right) = P\left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = P\left(\frac{5}{5}, \frac{15}{5}\right) = P(1, 3)$$

Hence the required point of division which divides the join of (–1, 7) and (4, – 3) in the ratio 2 : 3 is P(1, 3).

Ex.9 Find the co-ordinates of the points which divide the line segment joining A(–2, 2) and B(2, 8) into four equal parts. **[NCERT]**

Sol.



It is given that AB is divided into four equal parts : AP = PQ = QR = RB

Q is the mid-point of AB, then co-ordinates of Q are : $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right)$ = (0, 5)

P is the mid-point of AQ, then co-ordinates of P are: $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(\frac{-2}{2}, \frac{7}{2}\right) = \left(-1, \frac{7}{2}\right)$

Also, R is the mid-point of QB, then co-ordinates of R are: $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(\frac{2}{2}, \frac{13}{2}\right) = \left(1, \frac{13}{2}\right)$

Hence, required co-ordinates of the points are:

$P(-1, \frac{7}{2})$, Q(0, 5), $R(1, \frac{13}{2})$

Ex.10 If the point C(–1,2) divides the lines segment AB in the ratio 3 : 4, where the co-ordinates of A are (2, 5), find the coordinates of B.

Sol. Let C (–1, 2) divides the line joining A (2, 5) and B (x, y) in the ratio 3 : 4. Then,

$$C\left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = C(-1, 2)$$

$$\Rightarrow \frac{3x+8}{7} = -1 \ \& \ \frac{3y+20}{7} = 2$$

$$\Rightarrow 3x + 8 = -7 \ \& \ 3y + 20 = 14$$

$$\Rightarrow x = -5 \ \& \ y = -2$$

The coordinates of B are : B (–5, – 2)

Ex.11 Find the ratio in which the line segment joining the points (1, – 7) and (6, 4) is divided by x-axis.

Sol. Let C (x, 0) divides AB in the ratio k : 1.

By section formula, the coordinates of C are given by :

$$C\left(\frac{6k+1}{k+1}, \frac{4k-7}{k+1}\right)$$

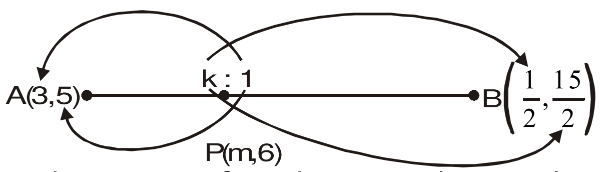
But C (x, 0) = $C\left(\frac{6k+1}{k+1}, \frac{4k-7}{k+1}\right)$

$$\Rightarrow \frac{4k-7}{k+1} = 0 \Rightarrow 4k - 7 = 0 \Rightarrow k = \frac{7}{4}$$

i.e., the x-axis divides AB in the ratio 7 : 4.

Ex.12 Find the value of m for which coordinates (3,5), (m,6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear.

Sol. Let P (m, 6) divides the line segment AB joining A (3,5), B $\left(\frac{1}{2}, \frac{15}{2}\right)$ in the ratio k : 1.



Applying section formula, we get the co-ordinates of P :

$$\left(\frac{\frac{1}{2}k+3\times 1}{k+1}, \frac{\frac{15}{2}k+5\times 1}{k+1}\right) = \left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right)$$

But P (m, 6) = $P\left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right)$

$\Rightarrow$ m = $\frac{k+6}{2(k+1)}$ and also $\frac{15k+10}{2(k+1)} = 6$

$\Rightarrow$ $\frac{15k+10}{2(k+1)} = 6 \Rightarrow 15k + 10 = 12(k + 1)$

$\Rightarrow$ $15k + 10 = 12k + 12$ $\Rightarrow$ $15k - 12k = 12 - 10$

$\Rightarrow$ $3k = 2$ $\Rightarrow$ $k = \frac{2}{3}$

Putting $k = \frac{2}{3}$ in the equation $m = \frac{k+6}{2(k+1)}$ we get :

$$m = \frac{\left(\frac{2}{3}+6\right)}{2\left(\frac{2}{3}+1\right)} = \frac{\left(\frac{2+18}{3}\right)}{2\left(\frac{2+3}{3}\right)}$$

$$= \frac{20}{3}\times\frac{3}{10} = \frac{20}{10} \quad \left(\because k = \frac{2}{3}\right) \quad m = \frac{10\times 2}{10} = 2$$

Required value of m is 2 $\Rightarrow m = 2$

Ex.13 The two opposite vertices of a square are (–1, 2) and (3, 2). Find the co-ordinates of the other two vertices.

Sol. Let ABCD be a square and two opposite vertices of it are A(–1, 2) and C(3, 2). ABCD is a square.

$\Rightarrow$ AB = BC $\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow$ $(x + 1)^2 + (y - 2)^2 = (x - 3)^2 + (y - 2)^2$

$\Rightarrow$ $x^2 + 2x + 1 = x^2 - 6x + 9$

$\Rightarrow$ $2x + 6x = 9 - 1 = 8 \Rightarrow 8x = 8 \Rightarrow x = 1$

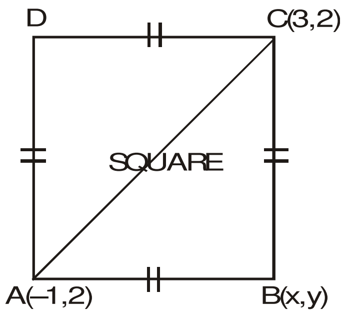
ABC is right Δ at B, then

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow$ $(3 + 1)^2 + (2 - 2)^2$

$= (x + 1)^2 + (y - 2)^2 + (x - 3)^2 + (y - 2)^2$

$\Rightarrow$ $16 = 2(y - 2)^2 + (1 + 1)^2 + (1 - 3)^2$



$\Rightarrow$ $16=2(y-2)^2+4+4 \Rightarrow 2(y-2)^2=16-8=8$

$\Rightarrow$ $(y - 2)^2 = 4 \Rightarrow y - 2 = \pm 2 \Rightarrow y = 4$ and 0

i.e. when x = 1 then y = 4 and 0

Co-ordinates of the opposite vertices are : B(1, 0) or D(1, 4)

Ex.14 The co-ordinates of the vertices of ΔABC are A(4, 1), B(−3, 2) and C(0, k). Given that the area of ΔABC is 12 unit^2. Find the value of k.

Sol. Area of ΔABC formed by the given-points A(4, 1), B(−3, 2) and C(0, k) is

$= \frac{1}{2} |\, 4\,(2 - k) + (-3)\,(k - 1) + 0\,(1 - 2)|$

$= \frac{1}{2} |8 - 4k - 3k + 3| = \frac{1}{2}(11 - 7k)$

But area of ΔABC = 12 unit^2 .... (given)

$\frac{1}{2}|11 - 7k| = 12$

$\Rightarrow |11 - 7k| = 24 \pm (11 - 7k) = 24$ $\Rightarrow$ $11 - 7k = 24$ or $-(11 - 7k) = 24$

$-7k = 24 - 11 = 13$

$\Rightarrow$ $k = -\frac{13}{7}$ or $-(11 - 7k) = 24$ $\Rightarrow$ $-11 + 7k = 24 \Rightarrow 7k = 24 + 11 = 35$

$\Rightarrow$ $k = \frac{35}{7} = 5$

Hence the values of k are : 5, $\frac{-13}{7}$.

Ex.15 Find the area of the quadrilateral whose vertices taken in order are (−4, −2), (−3, −5), (3, −2) and (2, 3).

Sol. Join A and C. **[NCERT]**

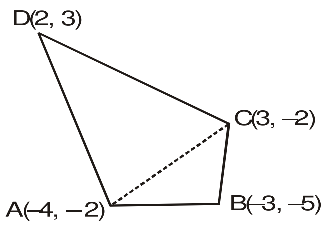
The given points are A(−4, −2), B(−3, −5), C(3, −2) and D(2, 3)

Area of ΔABC

$= \frac{1}{2}|(-4)\,(-5 + 2) - 3\,(-2 + 2) + 3\,(-2 + 5)|$

$= \frac{1}{2}|\,20 - 8 - 6 + 15|$

$= \frac{21}{2} = 10.5$ sq. units



Area of ΔACD

$= \frac{1}{2}|(-4)(-2 - 3) + 3(3 + 2) + 2\,(-2 + 2)|$

$= \frac{1}{2}|\,20 + 15| = \frac{35}{2} = 17.5$ sq. units.

Area of quadrilateral ABCD = ar. (ΔABC) + ar. (ΔACD) = (10.5 + 17.5) sq. units = 28 sq. units

Ex.16 Find the value of p for which the points (−1, 3), (2, p), (5, −1) are collinear.

Sol. The given points A (−1, 3), B (2, p), C (5, − 1) are collinear.

$\Rightarrow$ Area ΔABC formed by these points should be zero.

$\Rightarrow$ The area of $\Delta ABC = 0$

$\Rightarrow \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

$\Rightarrow -1(p+1) + 2(-1 - 3) + 5(3 - p) = 0$

$\Rightarrow -p - 1 - 8 + 15 - 5p = 0$

$\Rightarrow -6p + 15 - 9 = 0 \Rightarrow -6p = -6 \Rightarrow p = 1$

Hence the value of p is 1.

EXERCISE – I UNSOLVED PROBLEMS

Q.1 Find the points on the line through A(5, −4) and B(−3, 2) that are twice as far from A as from B.

Q.2 Find the ratio in which that line segment joining (2, −3) and (5, 6) is divides by x-axis.

Q.3 The three vertices of a rhombus, taken in order, area (2, −1), (3, 4) and (−2, 3). Find the fourth vertex.

Q.4 Find the vertices of a triangle, the mid- point of whose sides are (3, 1), (5, 6) and (−3, 2).

Q.5 Find the area of triangle formed by the points P(2, 1), Q(6, 1) and R(2, 4)

Q.6 Show that the points $P\left(\frac{-3}{2}, 3\right)$, Q(6, −2) and R(−3, 4) are collinear.

Q.7 P(2, 1), Q(4, 2), R(5, 4) and S(3, 3) are vertices of a quadrilateral, find the area of quadrilateral PQRS

Q.8 Find the distance between the points (3, − 1) & (− 2, 4)

Q.9 Show that the points A (a, a), B (− a, − a) and C $(-a\sqrt{3}, a\sqrt{3})$ form an equilateral triangle.

Q.10 Find the value of x such that PQ = QR, where P, Q and R are (6, − 1), (1, 3) and (x, 8) respectively.

Q.11 Find the co-ordinate of the point which divides the line segment joining the points (5, -2) and (9, 6) in the ratio 3:1.

Q.12 Find the ratio in which the point (2, y) divides the join (− 4 , 3) and (6, 3) and hence find the value of y.

Q.13 (i) Show that the points (1, 7), (4, 2) (−1, − 1) and (−4, 4) are the vertices of a square.

(ii) Show that the points A (5, 6), B(1, 5), C(2, 1) and D (6, 2) are the vertices of a square.

(iii) Show that the points A (1, 2), B(5, 4), C(3, 8) and D(−1, 6) are the vertices of a square.

Q.14 (i) Show that the points (0, −1), (−2, 3), (6, 7) and (8, 3) are the vertices of a rectangle. Also find the area of the rectangle.

(ii) Show that the points A(2, −2), B(14, 10), C(11, 13) and D(−1, 1) are the vertices of a rectangle.

Q.15 Name the type of quadrilateral formed by the following points and give reasons for your answer

(i) (−1, −2), (1, 0), (−1, 2), (−3, 0)

(ii) (4, 5), (7, 6), (4, 3), (1, 2)

Q.16 If two vertices of an equilateral triangle are (0, 0) and (3, 0), find the third vertex.

Q.17 The centre of a circle is C(2α − 1, 3α + 1) and it passes through the point A(−3, −1). If a diameter of the circle of length 20 units, find the value (s) of α.

Q.18 (i) Find the centre of a circle passing through the points (6, −6), (3, −7) and (3, 3). Also find its radius.

(ii) Find the coordinates of the point equidistant from the three given points

A(5, 1), B(−3, −7) and C(7, −1).

Q.19 (i) Find the ratio in which the line segment joining the points (6, 4) and (1, −7) is divided by x-axis.

(ii) Find the ratio in which the y-axis divides the line segment joining the points (5,−6) and (−1, −4). Also find the coordinates of the point of intersection.

(iii) In what ratio is the line segment joining the points (−2, −3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

Q.20 (i) Find the ratio in which the point (−3, p) divides the line segment joining the points (−5, −4) and (−2, 3). Hence find the value of p.

(ii) Determine the ratio in which the point (−6, a) divides the join of A(−3, −1)and B(−8, 9). Also find the value of a.

(iii) Find the ratio in which C(p, 1) divides the join of A(−4, 4) and B(6, −1) and hence find the value of p.

(iv) Find the ratio in which the point P whose ordinate is −3 divides the join of A(−2, 3) and $B\left(5, -\frac{15}{2}\right)$. Hence find the coordinates of P.

Q.21 (i) Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, −2) and B(3, 7). Also find the coordinates of the point of division.

(ii) In what ratio does the line $x - y - 2 = 0$ divide the line segment joining (3, −1) and (8, 9) ? Also find the coordinates of the point of division.

Q.22 (i) If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3), taken in order, are the vertices of a parallelogram, find the values of p.

(ii) If A(1, 2), B(4, q), C(p, 6) and D(3, 5), taken in order, are the vertices of parallelogram, find the values of p and q.

Q.23 If A(4, −6), B(3, −2) and C(5, 2) are the verticesof a triangle and D is mid-point of BC, find the coordinates of the point D. Also find the areas of Δs ABD and ACD. Hence verify that the median AD divides the triangle ABC two triangles of equal areas.

Q.24 A, B and C are the points (0, −1), (2, 1) and (0, 3) respectively, and D, E and F are mid-points of the sides BC, CA and AB respectively. Prove analytically that the area of ΔABC is 4 times the area of ΔDEF.

Q.25 If A(−5, 7), B(−4, −5), C(−1, −6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

Q.26 Find the area of the quadrilateral whose vertices taken in order, are

(i) (2, 1), (6, 0), (5, −2), (−3, −1)

(ii) (−4, −2), (−3, −5), (3, −2), (2, 3)

Q.27 Let A(4, 2), B(6, 5) and C(1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point G on AD such that AG : GD = 2 : 1

(iii) Find the areas of Δs GBC and ABC and verify that the area of ΔABC is 3 times the area of ΔGBC.

ANSWER KEY

1. (−11, 8). **2.** 2 : 1 **3.** (3, −2).

4. A(−1, 7), B(−5, −3) and C (11, 5).

5. 6 sq. units. **7.** 3 sq. units.

8 $5\sqrt{2}$ **10** x = 5, − 3 **11** (8, 4)

12 3:2, y = 3 **14.** (i) 40 sq. units

15. (i) square (ii) parallelogram

16. $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ or $\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$ **17.** $2, \frac{-46}{13}$

18. (i) (3,−2); 5 units (ii) (2, −4)

19. (i) 4 : 7 (ii) 5 : 1; $\left(0, \frac{-13}{3}\right)$ (iii) 2:3;(0, 1)

20. (i) 2 : 1, 2/3 (ii) 3:2; 5

(iii) 3:2; 2 (iv) 4 : 3; (2, −3)

21. (i) 2 : 9; $\left(\frac{24}{11}, \frac{-4}{11}\right)$ (ii) 2: 3; (5, 3)

22. (i) 7 (ii) p = 6; q = 3

23. (4, 0) ; 3 sq. unit, 3sq. unit

24. D(1, 2), E (0, 1), F(1, 0)

25. 72 sq. units

26. (i) 15 sq. unit (ii) 28 sq. unit

27. (i) $\left(\frac{7}{2}, \frac{9}{2}\right)$ (ii) $\left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) $\frac{13}{6}$ sq. units; $\frac{13}{2}$ sq. units

EXERCISE – II BOARD PROBLEMS

Q.1 Show that the point A(5, 6), B(1, 5) C(2, 1) and D(6, 2) are the vertices of a square. **[Delhi-2004]**

Q.2 Determine the ratio in which the point P(m, 6) divides the join of A(–4, 3) and B(2, 8). Also find the value of m.

OR

A(3, 2) and B(–2, 1) are two vertices of a triangle ABC, whose centroid G has coordinates $\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle. **[Delhi-2004]**

Q.3 Show that the points A(2, –2), B(14, 10), C(11, 13) and D(–1, 1) are the vertices of a rectangle. **[AI-2004]**

Q.4 Prove that the coordinates of the centroid of a ΔABC, with vertices. $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are given by $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ **[AI-2004]**

Q.5 Determine the ratio in which the point (–6, a) divides the join of A(–3, –1) and B(–8, 9). Also find the value of a. **[AI-2004]**

Q.6 Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, –3) **[AI-2004]**

Q.7 Prove that the points A(0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square. **[Foreign-2004]**

Q.8 Find the value of K, if the points A(8, 1), B(3, –4) and C(2, K) are collinear. **[AI-2010]**

Q.9 Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x – y + 2 = 0, find the value of K. **[AI-2010]**

Q.10 If the points (p, q), (m, n) and (p – m, q – n) are collinear, show that pn = qm. **[AI-2010]**

Q.11 The coordinates of the mid-point of the line joining the points (3p, 4) and (– 2, 2q) are (5, p). Find the values of p and q. **[Delhi 2004C]**

Q.12 Two vertices of a triangle are (1, 2) and (3, 5). If the centroid of the triangle is at the origin, find the coordinates of the third vertex.

OR

If 'a' is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin, find the coordinates of the vertices of the triangle ABC. **[Delhi 2004C]**

Q.13 Find the ratio in which the line-segment joining the points (6, 4) and (1, –7) is divided by x-axis. **[AI-2004C]**

OR

The coordinates of two vertices A and B of a triangle An are (1, 4) and (5, 3) respectively. If the coordinates of the centroid of ΔABC are (3, 3), find the coordinates of the third vertex C. **[AI-2004C]**

Q.14 Find the value of m for which the points with coordinates (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear. **[AI-2004C]**

Q.15 Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, –1); (1,3) and (x, 8) respectively.

OR

Find a point on x-axis which is equidistant from the points (7, 6) and (–3, 4). **[Delhi 2005]**

Q.16 The line-segment joining the points (3, –4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, –2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q. **[Delhi-2005]**

Q.17 Prove that the points (0, 0), (5, 5) and (–5, 5) are vertices of a right isosceles triangle.

OR

If the point P(x, y) is equidistant from the point A(5,1) and B(–1, 5), prove that 3x=2y. **[AI-2005]**

Q.18 The line joining the points (2, 1) and (5, –8) is trisected at the points P and Q. If point P lies on the line 2x – y + k = 0, find the value of k. **[AI-2005]**

Q.19 Show that the points (0, –1); (2, 1); (0, 3) and (–2, 1) are the vertices of a square.

OR

Find the value of K such that the point (0, 2) is equidistant from the points (3, K) and (K, 5). **[Foreign-2005]**

Q.20 The base BC of an equilateral ΔABC lies on y-axis. The coordinates of point C are (0, –3). If the origin is the mid-point of the base BC, find the coordinates of the points A and B. **[Foreign 2005]**

Q.21 Find the coordinates of the point equidistant from the points A(1, 2), B(3, – 4) and C(5,–6).

OR

Prove that the points A (–4, –1), B(–2, – 4), C(4, 0) and D (2, 3) are the vertices of a rectangle. **[Delhi-2005C]**

Q.22 Find the coordinates of the points which divide the line-segment joining the points (– 4,0) and (0, 6) in three equal parts. **[Delhi-2005C]**

Q.23 Two vertices of ΔABC are given by A(2, 3) and B(–2, 1) and its centroid is $G\left(1, \frac{2}{3}\right)$. Find the coordinates of the third vertex C of the ΔABC. **[AI-2005C]**

Q.24 Show that the points A(1, 2), B(5, 4), C(3, 8) and D (–1, 6) are the vertices of a square.

OR

Find the co-ordinates of the point equidistant from three given points A (5, 1), B(–3, –7) and C(7,–1). **[Delhi-2006]**

Q.25 Find the value of p for which the points (–1,3), (2, p) and (5,–1) are collinear. **[Delhi-2006]**

Q.26 If the points (10, 5), (8,4) and (6,6) are the mid· points of the sides of a triangle, find its vertices. **[Foreign-2006]**

Q.27 In what ratio is the line segment joining the points (–2, – 3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

OR

If A (5, – 1), B (– 3, – 2) and C (–1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid. **[Delhi-2006C]**

Q.28 If (–2, –1); (a, 0); (4, b) and (1,2) are the vertices of a parallelogram, find the values of a and b. **[AI-2006C]**

Q.29 Show that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle. **[Delhi-2007]**

Q.30 In what ratio does the lines x – y – 2 = 0 divides the line segment joining (3, –1) and (8, 9) ? **[Delhi-2007]**

Q.31 Three consecutive vertices of a parallelogram are (–2, 1); (1, 0) and (4, 3). Find the coordinates of the fourth vertex. **[AI-2007]**

Q.32 If the point C (– 1, 2) divides the line segment AB in the ratio 3 : 4, where the coordinates of A are (2, 5), find the coordinates of B. **[AI-2007]**

Q.33 For what value of p, are the points (2, 1), (p, –1) and (–1, 3) collinear? **[Delhi-2008]**

Q.34 Determine the ratio in which the line 3x + 4y – 9 = 0 divides the line segment joining the points (1, 3) and (2, 7). **[Delhi-2008]**

Q.35 If the distances of P(x, y) from the points A(3, 6) and B(– 3, 4) are equal, prove that 3x + y = 5. **[Delhi-2008]**

Q.36 For what value of p, the points (– 5, 1), (1, p) and (4, –2) are collinear? **[Delhi-2008]**

Q.37 For what value of k are the points (1, 1), (3, k) and (–1, 4) are collinear?

OR

Find the area of the ΔABC with vertices A(– 5, 7), B(– 4, – 5) and C(4, 5) **[AI-2008]**

Q.38 If the point P(x, y) is equidistant from the points A(3, 6) and B(–3, 4) prove that 3x + y – 5 = 0. **[AI-2008]**

Q.39 The point R divides the line segment AB, where A(–4, 0) and B(0, 6) such that $AR = \frac{3}{4}AB$. Find the co-ordinates of R. **[AI-2008]**

Q.40 The co-ordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB find co-ordinates of P such that $\frac{AP}{PB} = \frac{4}{3}$. **[AI-2008]**

Q.41 If A(4, – 8), B(3, 6) and C(5, –4) are the vertices of a ΔABC, D is the mid point of BC and P is a point on AD joined such that $\frac{AP}{PD} = 2$, find the co-ordinates of P. **[AI-2008]**

Q.42 Find the value of k if the points (k, 3), (6, – 2) and (– 3, 4) are collinear. **[Foreign-2008]**

Q.43 If P divides the join of A(– 2, – 2) and B(2, – 4) such that $\frac{AP}{AB} = \frac{3}{7}$, find the co-ordinates of P. **[Foreign-2008]**

Q.44 The mid points of the sides of a triangle are (3, 4), (4, 6) and (5, 7). Find the co-ordinates of the vertices of the triangle. **[Foreign-2008]**

Q.45 Show that A(–3, 2), B(–5, – 5), C(2, – 3) and D(4, 4) are the vertices of a rhombus. **[Foreign-2008]**

Q.46 Find the ratio in which the line 3x + y – 9 = 0 divides the line-segment joining the points (1, 3) and (2, 7). **[Foreign-2008]**

Q.47 Find the distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$. **[Delhi-2009]**

Q.48 Find the point on y-axis which is equidistant from the points (5, − 2) and (− 3, 2).

OR

The line segment joining the points A(2, 1) and B(5, − 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k. **[Delhi-2009]**

Q.49 If P(x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a} + \frac{y}{b} = 1$. **[Delhi-2009]**

Q.50 Find the point on x-axis which is equidistant from the points (2, − 5) and (− 2, 9) **[Delhi-2009]**

OR

The line segment joining the points P(3, 3), Q(6, − 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by $2x + y + k = 0$, find the value of k.

Q.51 If the points A(4, 3) and B(x, 5) are on the circle with the centre O(2, 3), find the value of x. **[AI-2009]**

Q.52 Find the ratio in which the point (2, y) divides the line segment joining the points A(−2, 2) and B(3, 7). Also find the value of y. **[AI-2009]**

Q.53 Find the area of the quadrilateral ABCD whose vertices are A(−4,−2), B(−3,−5), C(3,−2) and D(2, 3). **[AI-2009]**

Q.54 Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). **[AI-2009]**

Q.55 If the mid-point of the line segment joining the points P(6, b − 2) and Q(− 2, 4) is (2, − 3), find the value of b. **[Foreign-2009]**

Q.56 Show that the points (−2, 5), (3, − 4) and (7, 10) are the vertices of a right angled isosceles triangle.

OR

The centre of a circle is (2α − 1, 7) and it passes through the point (− 3, − 1). If the diameter of the circle is 20 units, then find the value(s) of α. **[Foreign-2009]**

Q.57 If C is a point lying on the line segment AB joining A(1, 1) and B(2, − 3) such that 3AC = CB, then find the co-ordinates of C. **[Foreign-2009]**

Q.58 Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear. **[Foreign-2009]**

Q.59 If the points (−2, 1), (a, b) and (4, − 1) are collinear and a − b = 1, then find the values of a and b. **[Foreign-2009]**

Q.60 Find the value of K, if the points A(7, − 2), B(5, 1) and C(3, 2K) are collinear. **[AI-2010]**

ANSWER KEY

2. 3 : 2, −2/5 or (4, −4) **5.** 3 : 2, 5

6. (−2, 0) **8.** −5 **9.** $\frac{2}{3}$ **11.** p = 4, q = 2

12. (−4, −7) or A(a/2, $\sqrt{3}a/2$), B(0, 0), C(a,0)

13. 4 : 7 or (3, 2) **14.** 2

15. 5 or −3 or (3, 0) **16.** p = $\frac{7}{3}$, q = 0

18. k = −8 **19.** OR k = 1

20. $(\pm 3\sqrt{3}, 0)$ and (0, 3) **21.** (11, 2)

22. $\left(\frac{-8}{3}, 2\right), \left(\frac{-4}{3}, 4\right)$ **23.** (3, −2) **24.** OR (2, −4)

25. p = 1 **26.** (4, 5), (8, 7), (12, 3)

27. 2 : 3, (0, 1) or $\sqrt{65}, \left(\frac{1}{3}, \frac{5}{3}\right)$ **28.** a = 1, b = 3

30. 2 : 3 **31.** (1, 2) **32.** (−5, −2)

33. p = 5 **34.** 6 : 25 **36.** −1

37. −2 or 53 sq. units **39.** $\left(-1, \frac{9}{2}\right)$

40. $\left(\frac{11}{7}, \frac{18}{7}\right)$ **41.** (4, −2) **42.** k = $-\frac{3}{2}$

43. $\left(\frac{-2}{7}, \frac{-20}{7}\right)$ **44.** (4, 5), (2, 3), (6, 9)

46. 3 : 4 **47.** 2 **48.** (0, −2) or −8

50. (−7, 0) or − 8 **51.** 2 **52.** 4 : 1, 6

53. 28 sq. unit **54.** 1 sq. unit **55.** −8

56. OR −4 or 2 **57.** $\left(\frac{5}{4}, 0\right)$ **58.** x + 3y = 7

59. a = 1, b = 0 **60.** 2

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 If the distance between the points (x, 2) and (3, −6) is 10 units, then positive value of x is
(A) 3 (B) 9
(C) 6 (D) 1

Q.2 The values of y for which the distance between the points (2, −3) and (10, y) is 10 units, are
(A) −3, 9 (B) 5, 1
(C) −5, 1 (D) −9, 3

Q.3 The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is
(A) $a^2 + b^2$ (B) $\sqrt{a^2+b^2}$
(C) a + b (D) $a^2 - b^2$

Q.4 If A and B are the points (−6, 7) and (−1, −5), then 2AB is equal to
(A) 238 (B) 13
(C) 169 (D) 26

Q.5 The coordinates of the point on x-axis equidistant from the points, (2, −5) and (−2, 9) are
(A) (4, 0) (B) (14, 0)
(C) (−7, 0) (D) (0, −7)

Q.6 The coordinates of the point on y-axis which is equidistant from the points, (4,−3) and (5, 2) are
(A) (0, 20) (B) (0, −23)
(C) $\left(0, \frac{2}{5}\right)$ (D) $\left(0, \frac{4}{5}\right)$

Q.7 If the distance between the points (4, y) and (1, 0) is 5 units, then y is equal to
(A) ±4 (B) 4
(C) −4 (D) 0

Q.8 If P (2, 2), Q(−4, −4) and R(5, −8) are the vertices of a ΔPQR, then length of median through R is
(A) $\sqrt{117}$ units (B) $\sqrt{85}$ units
(C) $\sqrt{113}$ units (D) $\sqrt{65}$ units

Q.9 The points (a, b + c), (b, c + a) and (c, a + b) are
(A) vertices of an equilateral triangle
(B) vertices of an isosceles triangle
(C) vertices of a right triangle
(D) collinear

Q.10 If the points (k, 2k), (3k, 3k) and (3, 1) are collinear, then value of k is
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $-\frac{1}{3}$ (D) $-\frac{2}{3}$

Q.11 If A (5, 3), B(11, −5) and C(12, y) are the vertices of a right angled triangle, right angled at C, then values of y are
(A) 2, 4 (B) −2, 4
(C) 2, −4 (D) −4, −2

Q.12 If the points A(x, 2), B(−3, −4) and C(7, −5) are collinear, then x is
(A) −63 (B) 63
(C) 60 (D) −60

Q.13 If the points (x, 2x), (−2, 6) and (3, 1) are collinear, then value of x is
(A) $\frac{3}{4}$ (B) $\frac{3}{5}$
(C) $\frac{5}{3}$ (D) $\frac{4}{3}$

Q.14 The area of the triangle formed by the points (a, b + c), (b, c + a), and (c, a + b) is
(A) 0 (B) a + b + c
(C) abc (D) $(a + b + c)^2$

Q.15 If the area of the triangle formed by the points $\left(x, \frac{4}{3}\right)$, (−2, 6) and (3, 1) is 5 sq units, then x is
(A) 3 (B) 5
(C) $\frac{2}{3}$ (D) $\frac{3}{5}$

Q.16 The coordinates of the centroid of the triangle whose vertices are (1, 3), (−2, 7) and (5, −3), are
(A) (4, 7) (B) $\left(\frac{4}{3}, \frac{7}{3}\right)$
(C) $\left(2, \frac{7}{2}\right)$ (D) $\left(\frac{8}{3}, \frac{7}{3}\right)$

Q.17 The coordinates of the consecutive vertices of a parallelogram are (1, 3), (−1, 2) and (2, 5). The coordinates of the fourth vertex are
(A) (6, 4) (B) (4, 6)
(C) (−2, 0) (D) (−4, −6)

Q.18 The centroid of a triangle is the point (1, 4), If its two vertices are (4, −3) and (−9, 7), then the third vertex is
(A) (−3, 4) (B) (7, 4)
(C) (4, 7) (D) (8, 8)

Q.19 The mid-point of the line segment joining the points (4, 7) and (2 −3) is
(A) (2, 4) (B) (1, 2)
(C) (3, 2) (D) $\left(2, \frac{4}{3}\right)$

Q.20 The mid point of segment AB is the point (4, 0). If the coordinates of A are (3, −2), then the coordinates of the point B are
(A) (5, 2) (B) (11, −2)
(C) (9, 2) (D) (9, −2)

Q.21 The line segment joining the points (−3, −4) and (1, −2) is divided by the y-axis in the ratio
(A) 1 : 3 (B) 2 : 3
(C) 3 : 1 (D) 3 : 2

Q.22 The ratio in which the point (1, 3) divides the line segment joining the points (−1, 7) and (4, −3) is
(A) 3 : 2 (B) 2 : 3
(C) −2 : 3 (D) 3 : −2

Q.23 The distance of the point (−3, 4) from the origin is
(A) 5 units (B) 7 units
(C) 25 units (D) 1 unit

Q.24 The distance of point (−3, −5) from the origin is
(A) −8 units (B) 8 units
(C) $\sqrt{34}$ units (D) 34 units

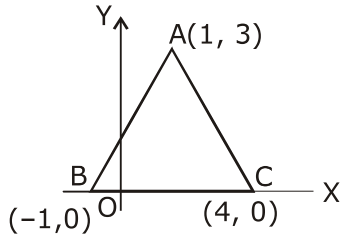
Q.25 The distance between the points P(−1, -5) and Q(−6, 7) is
(A) 12 units (B) 13 units
(C) 15 units (D) 169 units

Q.26 The distance between the points (a,b) and (−a,−b) is
(A) $2\sqrt{a^2+b^2}$ (B) $\sqrt{2}$ a
(C) 2 b (D) 2 | a |

Q.27 If the distance between the points (p, – 5) and (2, 7) is 13 units, then the values of p are
(A) 3, 7 (B) −3, 7
(C) 3, – 7 (D) −3, −7

Q.28 The area of a square whose vertices are A(0, −2, B(3, 1), C(0, 4) and D(−3, 1) is
(A) 18 sq. units (B) 15 sq. units
(C) $\sqrt{18}$ sq. units (D) $\sqrt{15}$ sq. units

Q.29 In the adjoining figure, the area of the triangle ABC is



(A) 15 sq. units (B) 10 sq. units
(C) 7.5 sq. units (D) 2.5 sq. units

Q.30 The point on the x-axis which is equidistant from the points A(−2, 3) and B(5, 4) is
(A) (0, 2) (B) (2, 0)
(C) (3, 0) (D) −2, 0)

Q.31 The point on the y-axis which is equidistant from the points A(−3, 4) and B(7, 6) is
(A) (15, 0) (B) (0, −15)
(C) (0, 15) (D) (0, 13)

Q.32 If A is an point on the y-axis whose ordinate is 5 and B is the point (−3, 1), then the length of AB is
(A) 8 units (B) 5 units
(C) 3 units (D) 25 units

Q.33 The coordinates of the mid-point of the line segment joining the points A(−4, −3) and B(2, 7) are
(A) (−2, −4) (B) (−2, 4)
(C) (2, 4) (D) (−1, 2)

Q.34 If the end points of a diameter of a circle are A(−2, 3) and B(4,−5), then the coordinates of its centre are
(A) (2, −2) (B) (1, −1)
(C) (−1, 1) (D) (−2, 2)

Q.35 If one end of a diameter of a circle is (2, 3) and the centre is (−2, 5) then the other end is
(A) (−6, 7) (B) (6, −7)
(C) (0, 8) (D) (0, 4)

Q.36 If the mid-point of the line segment joining the points P(a, b−2) and Q(−2, 4) is R(2, −3), then the values of a and b are
(A) a = 4, b= −5 (B) a = 6, b = 8
(C) a = 6, b = −8 (C) a = −6, b = 8

Q.37 The coordinates of the point which divides the join of$(-1, 7)$ and $(4, -3)$ in the ratio 2 : 3 internally are

(A) $(1, -3)$ (B) $(-1, 3)$
(C) $(-1, -3)$ (D) $(1, 3)$

Q.38 The ratio in which the point $C(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ is

(A) 2 : 7 (B) 7 : 2
(C) 2 : 5 (D) 5 : 2

Q.39 The ratio in which the line segment joining the points $(4, 6)$ and $(-7, -1)$ is divided by the x-axis is

(A) 1 : 6 (B) 4 : 7
(C) 7 : 4 (D) 6 : 1

Q.40 The centroid of the triangle whose vertices are $(3, -7)$, $(-8,6)$ and $(5, 10)$ is

(A) $(0, 9)$ (B) $(0, 3)$
(C) $(1, 3)$ (D) $(3, 3)$

Q.41 If the points $A(2, 3)$, $B(4,k)$ and $C(6, -3)$ are collinear, then the value of k is

(A) 1 (B) -1
(C) 0 (D) 3

Q.42 The area (in sq. units) of the triangle formed by the points $A(5, 2)$, $B(4, 7)$ and $C(7, -4)$ is

(A) -4 (B) 4
(C) -2 (D) 2

Q.43 In which quadrant does the point $(-3, 5)$ lie ?

(A) I (B) II
(C) III (D) IV

Q.44 In which quadrant does the point $(2, -4)$ lie ?

(A) I (B) II
(C) III (D) IV

Q.45 The distance of the point $P(3, 4)$ from the x-axis is

(A) 3 units (B) 4 units
(C) 7 units (D) 1 unit

Q.46 P is a point on x-axis at a distance of 3 units from y-axis to its right. The coordinates of P are

(A) $(3, 0)$ (B) $(0, 3)$
(C) $(3, 3)$ (D) $(-3, 3)$

Q.47 A is a point on y-axis at a distance of 4 units from x-axis, lying below x-axis. The coordinates of A are

(A) $(4, 0)$ (B) $(0, 4)$
(C) $(-4, 0)$ (D) $(0, -4)$

Q.48 The distance of the point $P(4, -3)$ from the origin is

(A) 1 unit (B) 3 units
(C) 5 units (D) 7 units

Q.49 The distance between the points $A(2, -3)$ and $B(2, 2)$ is

(A) 4 units (B) 5 units
(C) 3 units (D) 2 units

Q.50 What point on x-axis is equidistant from the points $A(7, 6)$ and $B(-3, 4)$?

(A) $(0, 4)$ (B) $(-4, 0)$
(C) $(3, 0)$ (D) $(0, 3)$

Q.51 The distance between the points $A(0, 5)$ and $B(-5, 0)$ is

(A) 5 units (B) $2\sqrt{5}$ units
(C) $\sqrt{10}$ units (D) $5\sqrt{2}$ units

Q.52 A point P divides the join of $A(5, -2)$ and $B(9, 6)$ in the ratio 3 : 1. The coordinates of P are

(A) $(4, 7)$ (B) $(8, 4)$
(C) $(12, 8)$ (D) $\left(\frac{11}{2}, 5\right)$

Q.53 In what ratio does the point $P(1, 2)$ divide the join of $A(-2, 1)$ and $B(7, 4)$?

(A) 1 :2 (B) 2 :1
(C) 3 :2 (D) 2 :3

Q.54 The point which divides the line segment joining the points $A(7, -6)$ and $B(3, 4)$ in the ratio 1 :2 lies in

(A) I quadrant (B) II quadrant
(C) III quadrant (D) IV quadrant

Q.55 In what ratio does the x-axis divide the join of $A(2\ -3)$ and $B(5, 6)$?

(A) 2 :3 (B) 3 :5
(C) 1 :2 (D) 2 : 1

Q.56 In what ratio does the y-axis divide the join of $P(-4, 2)$ and $Q(8, 3)$?

(A) 3 : 1 (B) 1 :3
(C) 2 : 1 (D) 1 :2

Q.57 The midpoint of the line segment joining the points $A(-2, 8)$ and $B(-6, -4)$ is

(A) $(4, 2)$ (B) $(-4, 2)$
(C) $(2, 6)$ (D) $(-4, -6)$

Q.58 If $P(-1, 1)$ is the midpoint of the line segment joining $A(-3, 6)$ and $B(1, b+ 4)$, then b = ?

(A) 1 (B) -1
(C) 2 (D) 0

Q.59 If P $\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(−6, 5) and B(−2, 3), then a = ?
(A) −4 (B) −12
(C) 12 (D) −6

Q.60 If the distance between the points A(2, −2) and B(−1, x) is 5, then
(A) x = −3 or x = 4 (B) x = 3 or x = −4
(C) x = −6 or x = 2 (D) x = 6 or x = −2

Q.61 The line 2x + y − 4 = 0 divides the line segment joining A(2, −2) and B(3, 7) in the ratio
(A) 2 : 5 (B) 2 : 9
(C) 2 : 7 (D) 2 : 3

Q.62 If A(4, 2), B(6, 5) and C(1, 4) be the vertices of ΔABC and AD is the median, then the coordinates of D are
(A) $\left(\frac{5}{2}, 3\right)$ (B) $\left(5, \frac{7}{2}\right)$
(C) $\left(\frac{7}{2}, \frac{9}{2}\right)$ (D) none of these

Q.63 If A(−1, 0), B(5, −2) and C(8, 2) are the vertices of a ΔABC, then its centroid is
(A) (12, 0) (B) (6, 0)
(C) (0, 6) (D) (4, 0)

Q.64 Two vertices of ΔABC are A(−1, 4) and B(5, 2) and its centroid is G(0, −3). Then, the coordinates of C are
(A) (4, 3) (B) (4, 15)
(C) (−4, −15) (D) (−15, −4)

Q.65 If the distance between the points A (4, p) and B(1, 0) is 5, then
(A) p = 4 only (B) p = −4 only
(C) p = ±4 (D) p = 0

Q.66 The three vertices of a parallelogram ABCD are A(−2, 3), B(6, 7) and C(8, 3). The fourth vertex D is
(A) (1, 0) (B) (0, 1)
(C) (−1 ,0) (D) (0, −1)

Q.67 The points A(−4, 0), B (4, 0) and C(0, 3) are the vertices of a triangle. which is
(A) isosceles (B) equilateral
(C) scalene (D) right-angled

Q.68 The points P(0, 6), Q(−5, 3) and R(3, 1) are the vertices of a triangle, which is
(A) equilateral (B) isosceles
(C) scalene (D) right -angled

Q.69 The points (a, a), (−a, −a) and $\left(-\sqrt{3}a, \sqrt{3}a\right)$ form the vertices of
(A) an equilateral triangle
(B) a scalene triangle
(C) an isosceles triangle
(D) a right triangle

Q.70 Three points A(1 − 2), B(3, 4) and C(4, 7) form
(A) a straight line
(B) an equilateral triangle
(C) a right-angled triangle
(D) none of these

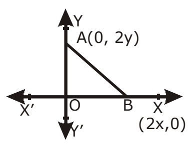
Q.71 If the points A(2, 3), B(5, k) and C(6, 7) are collinear, then
(A) k = 4 (B) k = 6
(C) $k = \frac{-3}{2}$ (D) $k = \frac{11}{4}$

Q.72 If the points A(1, 2), O(0, 0) and C(a, b) are collinear, then
(A) a = b (B) a = 2b
(C) 2a = b (D) a + b = 0

Q.73 The area of ΔABC with vertices A(a, b + c), B(b, c + a) and C(c, a + b) is
(A) $(a + b + c)^2$ (B) a + b + c
(C) abc (D) 0

Q.74 The point which lies on the perpendicular bisector of the line segment joining the points A(−2, −5) and B(2, 5) is
(A) (0, 0) (B) (0, 2)
(C) (2, 0) (D) (−2, 0)

Q.75 In the given figure A(0, 2y) and B(2x, 0) are the end points of line segment AB. The coordinates of point C which is equidistant from the three vertices of ΔAOB are



(A) (x, y) (B) (y, x)
(C) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (D) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Q.76 The points A(9, 0), B(9, 6), C(−9, 6) and D(−9, 0) are the vertices of a
(A) square (B) rectangle
(C) rhombus (D) trapezium

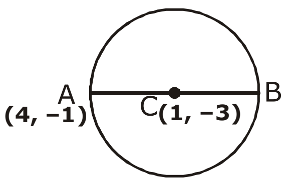
Q.77 The area of ΔABC with vertices A(3, 0), B(7, 0) and C(8, 4) is
(A) 14 sq units (B) 28 sq units
(C) 8 sq units (D) 6 sq units

Q.78 AOBC is a rectangle whose three vertices are A(0,3), O(0, 0) and B(5, 0). Length of each of its disgonals is
(A) 5 units (B) 3 units
(C) $\sqrt{34}$ units (D) 4 units

Q.79 A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, −5) is the midpoint of PQ, then the coordinates of P and Q are respectively
(A) (0, −5) and (2, 0)
(B) (0, −10) and (4, 0)
(C) (0, 10) and (−4, 0)
(D) (0, 4) and (−10 ,0)

Q.80 A circle drawn with origin as the centre passes through the point $A\left(\frac{13}{2}, 0\right)$. Which of the following points does not lie in the interior of the circle ?
(A) $\left(\frac{-3}{4}, 1\right)$ (B) $\left(2, \frac{7}{3}\right)$
(C) $\left(5, \frac{-1}{2}\right)$ (D) $\left(-6, \frac{5}{2}\right)$

Q.81 The coordinates of one end point of a diameter AB of a circle are A(4, −1) and the coordinates of the centre of the circle are C(1, −3). Then, the coordinates of B are



(A) (2, −5) (B) (−2, 5)
(C) (−2, −5) (D) (2, 5)

Q.82 The point on the x-axis which is equidistant from the points (5, 4) and (−2, 3) is **[NTSE]**
(A) (−2, 0) (B) (2, 0)
(C) (0, 2) (D) (2, 2)

Q.83 If the distances of p(x, y) from A(−1, 5) and B(5, 1) are equal, then **[NTSE]**
(A) 2x = y (B) 3x = 2y
(C) 3x = y (D) 2x = 3y

Q.84 If the point (x, y) is equidistant from the point (a + b, b − a) and (a − b, a + b), then which of the following is correct? **[NTSE]**
(A) ax = by (B) $ax^2 = by$
(C) ay = bx (D) $ay^2 = bx$

Q.85 Which of the following points is equidistant from (2, −3)? **[NTSE]**
(A) (−1, 0) (B) (1, 0)
(C) (−2, 0) (D) (2, 0)

Q.86 Which of the following point is equidistanat from (3, 2) and (−5, −2)? **[NTSE]**
(A) (0, 2) (B) (0, −2)
(C) (2, 0) (D) (2, −2)

Q.87 Which of the following points are the vertices of an equilateral triangle? **[NTSE]**
(A) (a, a), (−a, −a), (2a, a)
(B) (a, a) (−a, −a), $(-a\sqrt{3}, a\sqrt{3})$
(C) $(\sqrt{2}a, -a), (a, \sqrt{2}a), (a, -a)$
(D) (0, 0), (a, a), $(a, \sqrt{2}a)$

Q.88 If the points (−1, 3), (2, p) and (5, −1) are collinear, the value of p is **[NTSE]**
(A) 1 (B) −1
(C) 0 (D) $\sqrt{2}$

Q.89 The co-ordinates of the point which divides the line joining (1, −2) and (4, 7) internally in the ratio 1 : 2 are **[NTSE]**
(A) (1, 2) (B) (−1, −1)
(C) (−1, 2) (D) (2, 1)

Q.90 In what ratio is the line joining the points A(4, 4) and B(7, 7) divided by p(−1, −1)? **[NTSE]**
(A) 8 : 5 (B) 5 : 8
(C) 5 : 7 (D) 7 : 4

ANSWER KEY

1.	B	**2.**	D	**3.**	B	**4.**	D
5.	C	**6.**	C	**7.**	A	**8.**	B
9.	D	**10.**	C	**11.**	C	**12.**	A
13.	D	**14.**	A	**15.**	C	**16.**	B
17.	B	**18.**	D	**19.**	C	**20.**	A
21.	C	**22.**	B	**23.**	A	**24.**	C
25.	B	**26.**	D	**27.**	B	**28.**	A
29.	C	**30.**	B	**31.**	C	**32.**	B
33.	D	**34.**	B	**35.**	A	**36.**	C
37.	D	**38.**	A	**39.**	D	**40.**	B
41.	C	**42.**	D	**43.**	B	**44.**	D
45.	B	**46.**	A	**47.**	D	**48.**	C
49.	B	**50.**	C	**51.**	D	**52.**	B
53.	A	**54.**	D	**55.**	C	**56.**	D
57.	B	**58.**	B	**59.**	B	**60.**	C
61.	B	**62.**	C	**63.**	D	**64.**	C
65.	C	**66.**	D	**67.**	A	**68.**	D
69.	A	**70.**	A	**71.**	B	**72.**	C
73.	D	**74.**	A	**75.**	A	**76.**	B
77.	C	**78.**	C	**79.**	B	**80.**	D
81.	C	**82.**	B	**83.**	B	**84.**	C
85.	C	**86.**	B	**87.**	B	**88.**	A
89.	D	**90.**	B				